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AN APPROACH TO IDENTIFY THE INTERACTIONS BETWEEN THE CONTROL FACTORS IN A MAHALANOBIS-TAGUCHI SYSTEM

The Mahalanobis-Taguchi System (MTS) is, today, widely used to define the optimal conditions for the design stage of product development especially, in the field of Artificial Intelligence (AI) considering the non-linear properties and non-digital data. In this paper, an approach to identify the several interactions in a MTS is proposed. The MTS contains four methods; Mahalanobis-Taguchi (MT) method, Mahalanobis Taguchi Adjoint (MTA) method, Recognition Taguchi (RT) method and Taguchi (T) method. The method to use for the analysis is selected based on the system's properties. For the case of study used in this research, the unit space is created through the RT method and used to calculate the Mahalanobis-Taguchi distances (*MTD*). For the method proposed in this paper, the relationships between control factors and *MTDs* were firstly clarified by MTS (RT), then the same relationships were clarified using a modified design of experiments method, and the several interactions between control factors in MTS (RT) were finally identified by comparing the two relationships. Then effectiveness of the proposed method was evaluated by using a mathematical model.

1. INTRODUCTION

The Taguchi methods were first developed by incorporating the concept of error factors into the design of experiment [2–9], and currently consists of three components: static, dynamic, and MTS. In addition, the MTS consists of MT, MTA, RT, and T. These are now widely used especially in the industrial and medical fields [10–13]. On the other hand, the Internet of Things (IoT) and AI, developed as information and communication technologies, are today used in many devices and home appliances [14]. The "T" in IoT stands for Things (industrial products), which should be designed with the user's values, judgements, sensitivities and emotions in mind in order for the user to feel comfortable using them, and

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MTS is the perfect tool for these tasks. The AI mainly uses machine learning, neural networks, deep learning and MTS and it deals with big data that has become multidimensional and is providing efficient information. Among these, MTS is suitable for complex nonlinear processing contained in big data and is suitable for qualitative evaluation of human emotions and sensibility. The focus of this research is the MTS used in AI.

MTS is used as an analysis tool for complex nonlinear relationships with non-numeric data in the industrial field. The MTS is useful for a tool of analysis and prediction of complex nonlinear relationship in the case of fully automatic AI control, the main features are that it can be used as a black box regarding complex, nonlinear, and interaction-laden causal relationships, and it can be controlled with high accuracy, which is widely used in industry [15–20]. However, when MTS is used for anything other than fully automated AI control, the several interactions between the control factors in the MTS make highly accurate control difficult. On the other hand, if these interactions are clarified and actively incorporated into the control, high-precision control can be achieved and production efficiency can be improved.

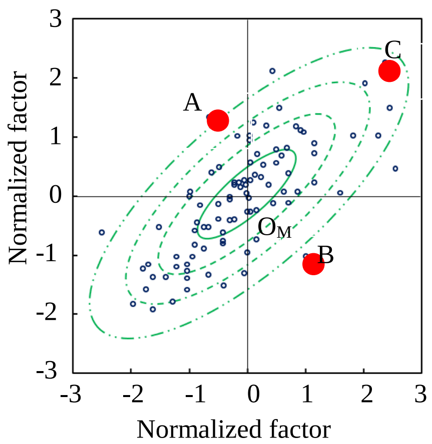
Therefore, in this study, a method for determining the interaction in MTS is proposed and evaluated as follows; the optimal conditions identification program [1] is used to determine the presence or absence of interactions in the MTS, find the control factors involved in the existing interactions and determine the degree of interactions. Then, the proposed method are evaluated by using a mathematical model. In this study, only the interaction between the control factors is considered, however the same approach can be adapted when there is a synergistic effect between the control factors. In this case, the RT method within MTS was used, however the proposed method can be used in other ways within MTS as well.

2. AN APPROACH TO IDENTIFY THE INTERACTIONS IN MTS USING THE OPTIMAL CONDITIONS IDENTIFICATION PROGRAM

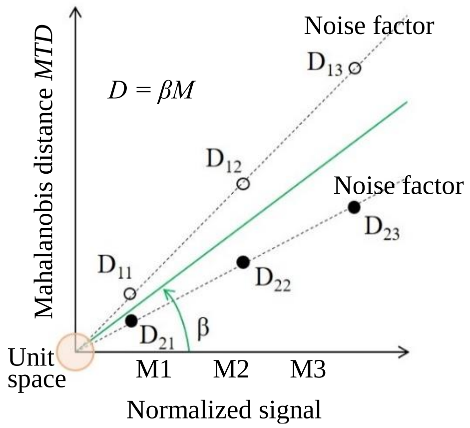
2. 1. BRIEF INSTRUCTIONS FOR MTS

The MTS evaluates the results of a multivariate analysis in Mahalanobis space by SN (signal-to-noise) ratio [21], [22]. The quantification of deviations from the centre in a multidimensional space can be used to classify things that cannot be classified using normal judgement and to identify influencing factors. Figure 1a shows a normalized plot of the two factors with the same distance of Mahalanobis surrounded by an ellipse. Points A and B are closer than point C in physical distance from the central point, but farther in *MTD*, and are distinguished from each other. As shown in Fig. 1b, the unit space (the criterion; the measure for the *MTD*) is firstly calculated, and then the MTD with noise factors is calculated, taking into account the influence of the error factor. Then the difference between the object and the unit space is quantitatively assessed by the length of the *MTD* [21], [22].

Here, the causality of the MTD_{ABC} (final properties) of the three control factors A (their levels A_x), B (their levels B_y) and C (their levels C_z), is expressed in MTS by equation (1).



(a) Relationship between both normalized factors and these Mahalanobis' distances



(b) Relationship between the Mahalanobis' distance and the normalized signal

Fig. 1. Schematic view for explanation regarding the MTS [21]

$$MTD_{ABC} = f(A_x) + g(B_y) + h(C_z) + i_{ab}(A_x, B_y) + i_{ac}(A_x, C_z) + i_{bc}(B_y, C_z) + i_{abc}(A_x, B_y, C_z) \quad (1)$$

If there is no interaction between the control factors, then eq. (1) is replaced by equation (2).

$$MTD_{ABC} = f'(A_x) + g'(B_y) + h'(C_z) \quad (2)$$

These functions, $f'(Ax)$, $g'(B_y)$ and $h'(C_z)$, which also include sigmoid functions, allow for the representation of non-linear and complex causal relationships.

2.2. BRIEF INSTRUCTIONS FOR THE OPTIMAL CONDITIONS IDENTIFICATION PROGRAM

In this section, the optimal conditions identification program used for the proposed method is explained.

The Design of Experiments (DOE) using a small number of experiments or CAE analyses is commonly used to estimate an optimum parameter combination in the new designs. The control factors (A to C) and their levels (A_1 to A_3, B_1 to B_3 and C_1 to C_3,) are shown in Table 1. The orthogonal table is used to set up the control factors and their levels in Table 1, as shown in Table 2.

The experiments are then carried out according to the numbers in the orthogonal table. The results are also given in Table 2 as final properties. From the principle of orthogonal tables, the relationship between the influence E of each control factor and the final property value P is shown in equation (3).

Finally, the final property values can be estimated on the basis of the additivity of the orthogonal sequences, which is the most important feature of the design of experiments. Therefore, the relationship between the influence E of each control factor and the all final property values $P_{Ax \cdot By \cdot Cz}$ can be estimated by equation (4).

Table 1. Control factors in an experimental design

Control factors			
Name	A	B	C
Levels	A_1	B_1	C_1
	A_2	B_2	C_2
	A_3	B_3	C_3

Table 2. Orthogonal array and final properties in an experimental design

	Control factors			Final properties
No.	A	B	C	
1	A_1	B_1	C_1	$P_{A1\cdot B1\cdot C1}$
2	A_1	B_2	C_2	$P_{A1\cdot B2\cdot C2}$
3	A_1	B_3	C_3	$P_{A1\cdot B3\cdot C3}$
4	A_2	B_1	C_2	$P_{A2\cdot B1\cdot C2}$
5	A_2	B_2	C_3	$P_{A2\cdot B2\cdot C3}$
6	A_2	B_3	C_1	$P_{A2\cdot B3\cdot C1}$
7	A_3	B_1	C_3	$P_{A3\cdot B1\cdot C3}$
8	A_3	B_2	C_1	$P_{A3\cdot B2\cdot C1}$
9	A_3	B_3	C_2	$P_{A3\cdot B3\cdot C2}$

$$\left.\begin{aligned}
E_{A1} &= (P_{A1\cdot B1\cdot C1} + P_{A1\cdot B2\cdot C2} + P_{A1\cdot B3\cdot C3}) / 3\\
E_{A2} &= (P_{A2\cdot B1\cdot C2} + P_{A2\cdot B2\cdot C3} + P_{A2\cdot B3\cdot C1}) / 3\\
E_{A3} &= (P_{A3\cdot B1\cdot C3} + P_{A3\cdot B2\cdot C1} + P_{A3\cdot B3\cdot C2}) / 3\\
E_{B1} &= (P_{A1\cdot B1\cdot C1} + P_{A2\cdot B1\cdot C2} + P_{A3\cdot B1\cdot C3}) / 3\\
E_{B2} &= (P_{A1\cdot B2\cdot C2} + P_{A2\cdot B2\cdot C3} + P_{A3\cdot B2\cdot C1}) / 3\\
E_{B3} &= (P_{A1\cdot B3\cdot C3} + P_{A2\cdot B3\cdot C1} + P_{A3\cdot B3\cdot C2}) / 3\\
E_{C1} &= (P_{A1\cdot B1\cdot C1} + P_{A2\cdot B3\cdot C1} + P_{A3\cdot B2\cdot C1}) / 3\\
E_{C2} &= (P_{A1\cdot B2\cdot C2} + P_{A2\cdot B1\cdot C2} + P_{A3\cdot B3\cdot C2}) / 3\\
E_{C3} &= (P_{A1\cdot B3\cdot C3} + P_{A2\cdot B2\cdot C3} + P_{A3\cdot B1\cdot C3}) / 3
\end{aligned}\right\} \quad (3)$$

$$P_{Ax\cdot By\cdot Cz} = E_{Ax} + E_{By} + E_{Cz} - (3-1)\, P_{ave} \quad (4)$$

Here, P_{ave} is the average of the final property values (the average of the final property values shown in Table 2, constant value). In design of experiments, the additivity of the orthogonal sequences can be used to estimate all combinations results (27 different results in this case) from a small number of experimental results.

The additivity (Equation (4)) of the orthogonal sequences was used for the optimal conditions identification program. In the previous explanations, all control factors had three levels. By increasing the number of these levels, the accuracy of the causal relationship increases, however it requires a long working time and a large cost. The relationship between the influence E (E_{Ax}, E_{By}, E_{Cz}) of each control factor and the each level (A_x, B_y and C_z) of each control factor was then displayed as three curves ($f(A_x)$, $g(B_y)$, $h(C_z)$) by curve fitting [1], [23].

In this way, the influence of successive level values can be processed quickly. Equation (4) is accordingly rewritten as equation (5).

$$P_{\mathrm{Ax \cdot Bx \cdot Cz}} = f''(A_x) + g''(B_y) + h''(C_z) - (3-1)\,P_{ave} \tag{5}$$

Instead of using the three level values of a control factor as data, as in Table 1, it is now possible to use continuous level value data by means of a function, which allows the final property values calculated by equation (4) to be interpolated quickly and easily. This is the optimal conditions identification program, which was used for the proposed method. In this study, the final properties are calculated using the unit spaces by MTS, so the final properties are replaced by the *MTD* and equation (5) becomes equation (6).

$$MTD_{\mathrm{ABC}} = f''(A_x) + g''(B_y) + h''(C_z) - (3-1)\,P_{ave} \tag{6}$$

If there is an interaction between the control factors, the additivity in the orthogonal table cannot be supported, and the causal relationship between the control factors and the *MTD* cannot be calculated accurately.

2.3. THE APPROACH USED TO IDENTIFY THE INTERACTIONS

In this section it is explained that the interaction of the control factors within MTS can be explored using the optimal conditions identification program [1]. The *MTD* can be calculated using either equation (1) or equation (2) in Section 2.1 by MTS. Equation (1) is the case where there is an interaction between the control factors and equation (2) is the case where there is no interaction between the control factors. On the other hand, the *MTD* is also calculated by equation (6) in Section 2.2 by the optimal conditions identification program.

The results obtained by the MTS (RT method) in the procedure (1) and the results obtained by the optimal conditions identification program in the procedure (3) are compared, so that the presence or absence of an interaction in the MTS (RT method) is explored, the extent of the interaction is clarified, and the controlling factors causing the interaction are identified. When the relationship between each control factor and the *MTD* is graphed and compared using the two equations for the *MTD*, it is possible to determine whether there is an interaction between the control factors. If there is no interaction, the two *MTD*s coincide. However, if there is an interaction between the control factors, the two *MTD*s differ, the difference is the part of the level at which the interaction is occurring, and the size of the difference is the size of the interaction.

The procedure of the method proposed in this research is as follows:

(1) The unit space of the *MTD* using the MTS (RT method) is defined. This unit space is the measure of *MTD.*

(2) The *MTD* is calculated by the unit space of (1) and evaluated for the target. So far, this is the general use of MTS. After this, the search for interactions in the unit space is carried out.

(3) The *MTD* was calculated by the optimal conditions identification program with the previous unit space, then target is evaluated by the *MTD* (using equation (6)) instead of the final property *P*.

(4) The results obtained with the MTS (RT method) in (1) and the results obtained with the optimal conditions identification program in (3) are compared, so that the presence or absence of an interaction in the MTS (RT method) is explored, the extent of the interaction is clarified, and the controlling factors causing the interaction are identified.

3. INTRODUCTION AND EVALUATION OF THE PROPOSED METHOD USING RANDOMLY GENERATED DATA

In this chapter, the proposed judgement method is explained in detail using randomly generated data. It is not real data of physical phenomena or human interactions. The MTR is evaluated using the RT method, and the interactions between the control factors are evaluated using the optimal conditions identification program.

3. 1. EXAMPLE OF DEFINITION AND EVALUATION OF A UNIT SPACE USING THE RT METHOD

The control factors C_1, C_2, C_3, C_4, and C_5 of Data I in Table 3 are first used to define for the unit space and the evaluation. The values assigned to each control factor are randomly selected within the defined ranges. The virtual ten persons' data for the unit space and five persons' data for the evaluation are defined respectively, according to the conditions in Table 3. Mr. Suzuki's software [24] was applied for RT method.

Table 3. The control factors of Data I and their ranges

Data I	Data for unit space					
	Control factors	C_1	C_2	C_3	C_4	C_5
	Range	$8.0 < C_1 < 10.0$	$80 < C_2 < 100$	$800 < C_3 < 1000$	$1800 < C_4 < 2000$	$0.08 < C_5 < 0.1$
	Data for evaluation					
	Control factors	C_1	C_2	C_3	C_4	C_5
	Range	$0.1 < C_1 < 10.0$	$1 < C_2 < 100$	$10 < C_3 < 1000$	$180 < C_4 < 2000$	$0.01 < C_5 < 0.2$

3.2. WRITING THE ALGEBRAIC EXPRESSION OF THE UNIT SPACE USING THE OPTIMAL CONDITIONS IDENTIFICATION PROGRAM

The unit space is defined as an algebraic expression using the unit space data and the optimal conditions identification program. In the design of experiments, the control factors previously introduced in Table 4 are used.

The orthogonal array L16, consisting of five control factors and four level values, is arranged by dividing the minimum and maximum values into four equal parts. Table 5 shows the MTD_{RT} calculation results as the final property value using the unit space data defined by the RT method. When this method is used in a concrete example, Table 4 use the real experiment data, however Table 5 always use the MTD_{RT} calculation results. Then, by applying the optimal conditions identification program [1], additivity and curve fitting to the results (using eq. (3), eq. (4) and curve fit technique), equation (7) is obtained.

Table 4. Calculation of MTD_{RT} for the Unit space and for the evaluation of Data I using RT method

	Unit space Data						Unit space distances MTD_{RT}
	Control factors	C_1	C_2	C_3	C_4	C_5	
Data I	Mr. AI	8.8	98.6	907.7	1843.2	0.085	0.22
	Mr. BI	8.4	80.0	916.5	1818.6	0.091	0.22
	Mr. CI	8.9	99.8	921.6	1813.7	0.090	0.24
	Ms. DI	9.3	90.9	947.1	1932.4	0.091	0.25
	Ms. EI	8.5	91.4	917.8	1895.6	0.093	0.21
	Mr. FI	8.8	99.7	940.7	1996.9	0.089	0.29
	Ms. GI	9.9	90.5	929.6	1955.5	0.093	0.17
	MS. H	8.7	81.2	860.3	1911.5	0.088	0.43
	Mr. I	9.6	96.4	988.2	1951.3	0.091	0.25
	Mr. J	9.6	97.4	916.6	1842.1	0.086	0.18
	Data for evaluation						Mahalanobis distances MTD_{RT}
	Control factors	C_1	C_2	C_3	C_4	C_5	
	Mr. SI	2.6	3.0	333.3	373.8	0.165	4.93
	Mr. TI	1.9	68.1	761.4	196.6	0.077	7.68
	Ms. MI	6.9	69.4	606.8	561.3	0.022	4.88
	Ms. O	5.2	6.0	439.3	275.6	0.012	5.55
	Mr. Q	2.9	38.8	880.1	1527.6	0.140	1.49

Table 5. Orthogonal array L16 and MTD_{RT} of the Data I

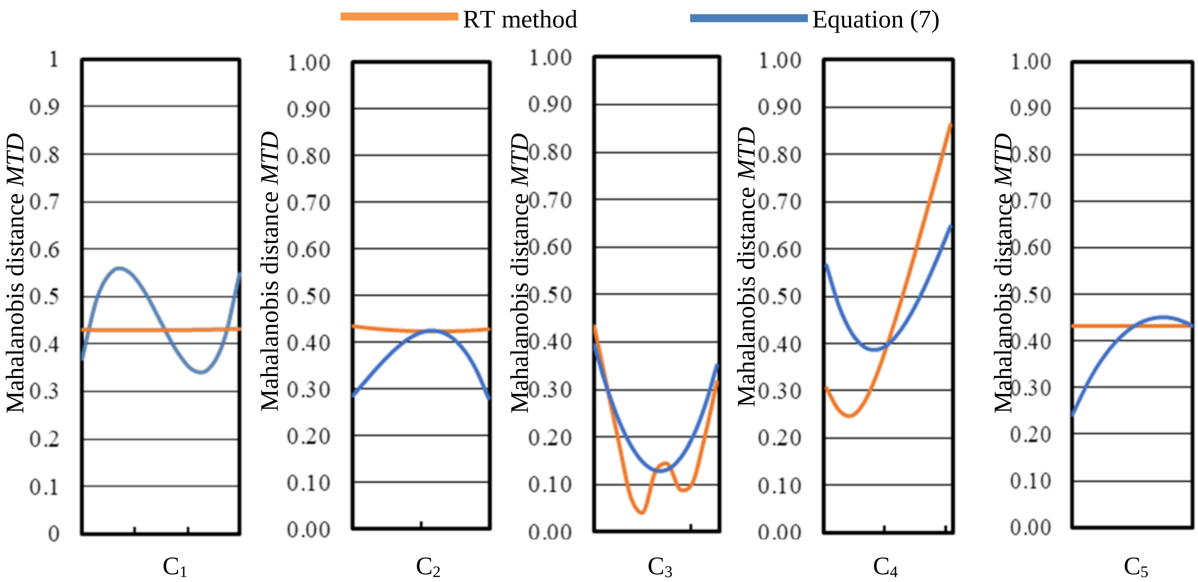
	No.	Control factors					MTD_{RT}
		C_1	C_2	C_3	C_4	C_5	
Orthogonal array L16	No. 1	8.4	80	860.3	1813.7	0.085	0.31
	No. 2	8.4	86.6	902.9	1874.8	0.087	0.18
	No. 3	8.4	93.2	945.6	1935.9	0.09	0.28
	No. 4	8.4	99.8	988.2	1996.9	0.093	0.37
	No. 5	8.9	80	902.9	1935.9	0.093	0.17
	No. 6	8.9	86.6	860.3	1996.9	0.09	0.86
	No. 7	8.9	93.2	988.2	1813.7	0.087	0.74
	No. 8	8.9	99.8	945.6	1874.8	0.085	0.05
	No. 9	9.4	80	945.6	1996.9	0.087	0.3
	No. 10	9.4	86.6	988.2	1935.9	0.085	0.25
	No. 11	9.4	93.2	860.3	1874.8	0.093	0.28
	No. 12	9.4	99.8	902.9	1813.7	0.09	0.26
	No. 13	9.9	80	988.2	1874.8	0.09	0.46
	No. 14	9.9	86.6	945.6	1813.7	0.093	0.37
	No. 15	9.9	93.2	902.9	1996.9	0.085	0.48
	No. 16	9.9	99.8	860.3	1935.9	0.087	0.54
						*MTD*s average μ_I	0.37

$$MTD\text{eq}(1) = \{f\, I\,(C_1) + g\, I\,(C_2) + h\, I\,(C_3) + i\, I\,(C_4) + m\, I\,(C_5) - (5-1)\,\mu\, I\}$$
$$= 0.966666824\, C_{13} – 26.51500432 C_{12} + 242.0038728 C_1 – 5.507322994\times10–5 C_{23} + 1.344714360\times10–2 C_{22} – 1.077344031 C_2 + 5.356439964\times10–8 C_{33} – 8.935365092\times10–5 C32 + 2.734862887\times10–2 C_3 – 8.742312616\times10–8 C_{43} + 5.245161991\times10–4 C_{42} – 1.045455643 C_4 + 5.486140434\times105 C_{53} – 1.588202275\times105 C_{52} + 1.522763216\times104 C_5 – 489.2196429 \quad (7)$$

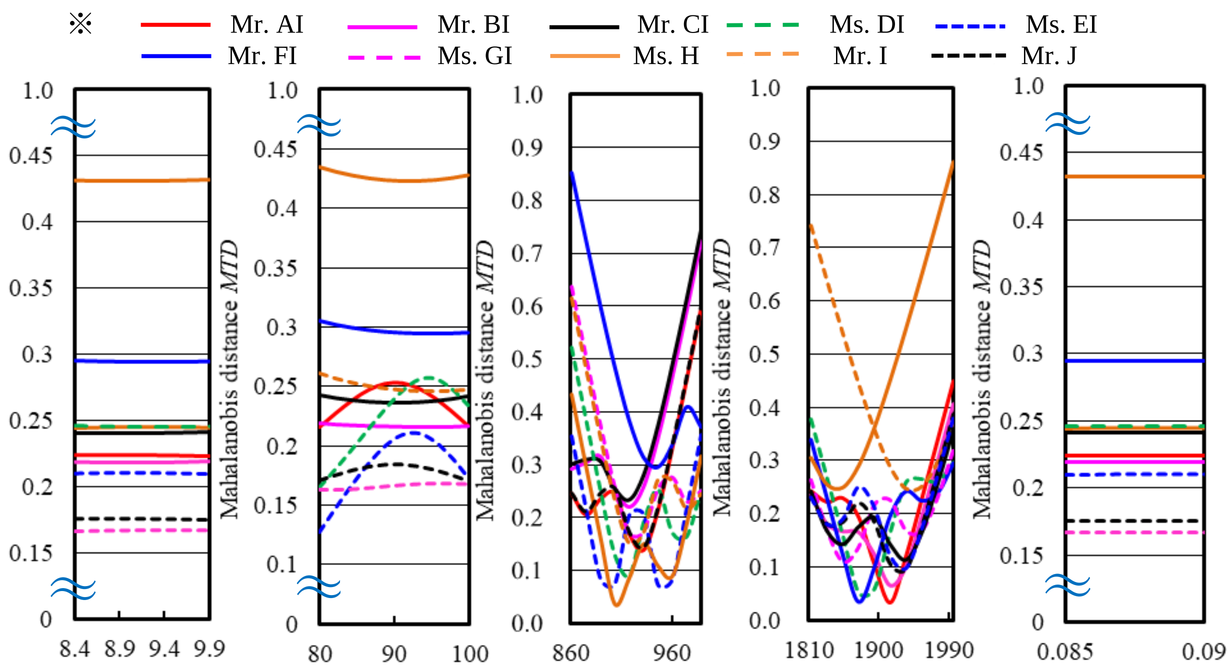
In this equation, $MTD_{eq(7)}$ is the independent variable and the control factors are the dependent variables. As it can be seen from the right side of equation (7), $MTD_{eq(7)}$ is calculated assuming that there is no interaction between the control factors in the unit space. There are five types of approximation in the optimal conditions identification program: linear, polynomial, logarithmic, exponential, and power approximation. In the current equation, the terms of all control factors are three-dimensional. Increasing the number of values of control factors makes the nonlinear correspondence more accurate.

3.3. DETERMINATION OF THE INTERACTIONS USING THE UNIT SPACE AND THE ALGEBRAIC EXPRESSION CREATED BY THE OPTIMAL CONDITIONS IDENTIFICATION PROGRAM

Figure 2a is used to explain the interaction between MTD_{RT} and $MTD_{eq(7)}$ and Fig. 2b is used to observe the behaviour of the data of 10 persons. The effect of each level value on the MTD_{RT} and the control factors can be visually observed from Fig. 2a. It is noticed that the fluctuations in the level values of C_1, C_2, and C_5 do not affect the MTD_{RT}, while the level values of the control factors C_3 and C_4 have a large non-linear effect on MTD_{RT}. These considerations provide useful information when managing data I. It can be assumed that all the control factors C_1, C_2, C_3, C_4, and C_5 are interacting with each other by comparing MTD_{RT} and MTDeq (7) (Fig. 2).



(a) Graphic Representation of Ms. H's Data



(b) Graphic representation of the 10 persons' data used in Table 2

Fig. 2. Comparison of *MTD* using the RT method and using the algebraic expression

For instance, the control factors C_3 and C_4 have clearly a significant and complex interaction. The *MTD*s tend to be qualitatively similar to the data variance of the 10 persons for the unit space from Fig. 2b, however they are quantitatively different, and the interaction is non-linear and quite complex. Therefore, it is concluded that it is necessary to inspect all the data in order to identify the different types of interactions.

4. EVALUATION OF THE PROPOSED METHOD USING A MATHEMATICAL MODEL

4.1. DETERMINATION OF THE PRESENCE OR ABSENCE OF INTERACTIONS AND IDENTIFICATION OF THE CONTROL FACTORS INVOLVED IN THE INTERACTIONS

A mathematical model given by the equation (8) with an intentional interaction which instead of using the unit space is used to determine the presence or absence of interactions, identify the controlling factors involved in the interactions and analyse the effect of the interactions.

$$MTD_{\mathrm{MATH}} = (90C_1^4) + (5C_2^2) + (100C_3) + (40C_4) + (0C_5) + 0.24(3.5C_1 \times C_2)^2 + 0.007(5C_1 \times 0.13C_3)^2 \div 3400000 \quad (8)$$

The first five terms on the right side of equation (8) show the effects of the five regulators on the *MTD*. The last two terms show respectively the interactions of the control factors C_1 and C_2 and the interaction of C_1 and C_3. The division is meant to reduce MTD_{MATH}

value to 1.0 or less. The aim of using the MTD_{MATH} is to know whether or not the interactions of the 6th and 7th terms on the right side can be determined by the proposed method.

The same random values from Table 1 are used to define the unit space in Table 6. MTD_{MATH} is calculated using those values and equation (8). The calculation of the interaction's percentage of the 6th and 7th terms on the right side of the MTD_{MATH} when the data No.1, No.2 and No.3 are substituted into equation (8), shows that the interaction of C_1 and C_2 is significant while the influence of C_3's interaction is small.

Table 6. Evaluation of the proposed method using equation (8) for other MTD_{MATH} calculations

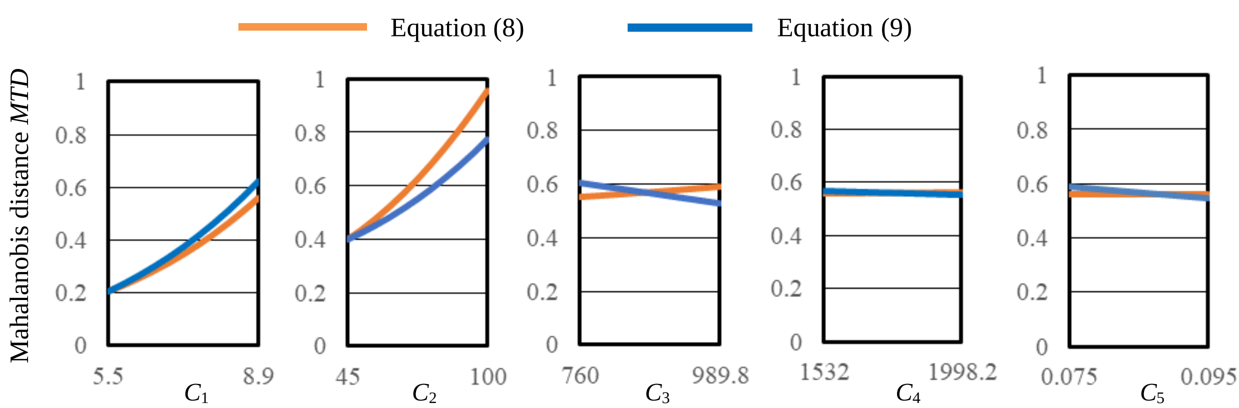
Data II	Data for unit space						Unit space distances MTD_{MATH}	Unit space distances $MTD_{\mathrm{eq(9)}}$	Unit space distances $MTD_{\mathrm{eq(10)}}$	Unit space distances $MTD_{\mathrm{eq(11)}}$
	Control factors	C_1	C_2	C_3	C_4	C_5				
	No. 1	8.9	66	820	1789	0.08	0.56	0.68	0.37	0.61
	No. 2	5.5	99.9	989.8	1998.2	0.075	0.38	0.44	0.21	0.45
	No. 3	8	45	760	1532	0.095	0.3	0.39	0.21	0.35

Interaction percentages	C_1			C_2	C_3	C_4	C_5
	Total	C_1 & C_2	C_1 & C_3	C_2 & C_1	C_3 & C_1		
No. 1	61%	53%	8%	53%	8%	Nothing	Nothing
No. 2	76%	69%	7%	69%	7%	Nothing	Nothing
No. 3	49%	38%	11%	38%	11%	Nothing	Nothing

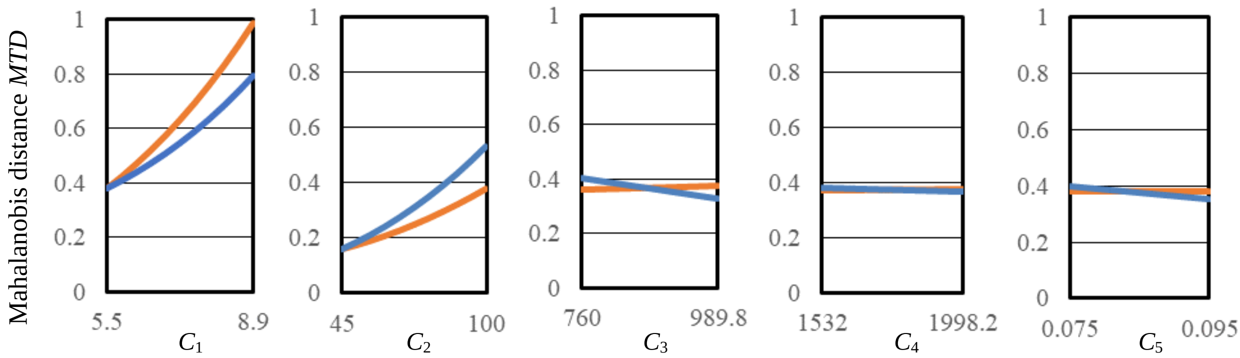
As in section 3.2, an experiment design using Data II (Table 6) and the optimal conditions identification program are used to obtain equation (9).

$$MTD_{\mathrm{eq(9)}} = 7.620882567\times10^{-4}C_1^3 - 2.569716203\times10^{-3}C_1^2 + 3.825670464\times10^{-2}C_1 + 1.031164108\times10^{-7}C_2^3 + 2.526112708\times10^{-5}C_2^2 + 1.45654337\times10^{-3}C_2 - 3.286952689\times10^{-4}C_3 - 2.546250193\times10^{-5}C_4 - 2.195454786C_5 + 0.210064 \quad (9)$$

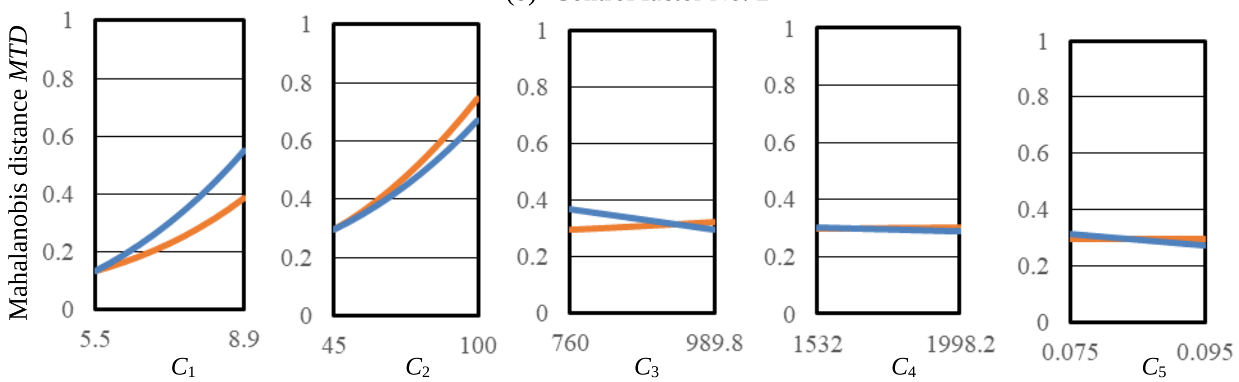
The similarity of $MTD_{\mathrm{eq(9)}}$ and $MTD_{\mathrm{eq(9)}}$ in Fig. 3, makes it possible to obtain an algebraic expression with $MTD_{\mathrm{eq\,(9)}}$ as a dependent variable and the control factors C_1, C_2, C_3, C_4, and C_5 as independent variables. In this algebraic expression, as it can be seen from the right side of equation (9), the control factors do not have any interaction with each other.



(a) Control factor No. 1



(b) Control factor No. 2



(c) Control factor No. 3

Fig. 3. Comparison of $MTD_{eq(8)}$ and $MTD_{eq(9)}$

It is deduced that C_4 and C_5 have no mutual interactions while C_1 and C_2 are interacting with each other from Table 7. It is still difficult to determine the presence or absence of the interaction for C_3.

Equation (9) is affected by the average value of the final properties $P_{C1\cdot C2\cdot C3\cdot C4\cdot C5}$ (in this case, the average value of $MTD_{eq(9)}$) in the relationship between $MTD_{eq(9)}$ and each control factor (C_1, C_2, C_3, C_4, and C_5), because the additivity of the orthogonal sequences in the optimal conditions identification program is used in the calculation process, and this effect also indirectly affects the control factors C_4 and C_5, which have no interaction. The differences between MTD_{MATH} using equation (8) and $MTD_{eq(9)}$ using equation (9) in Fig. 2 were occurred because of this reason.

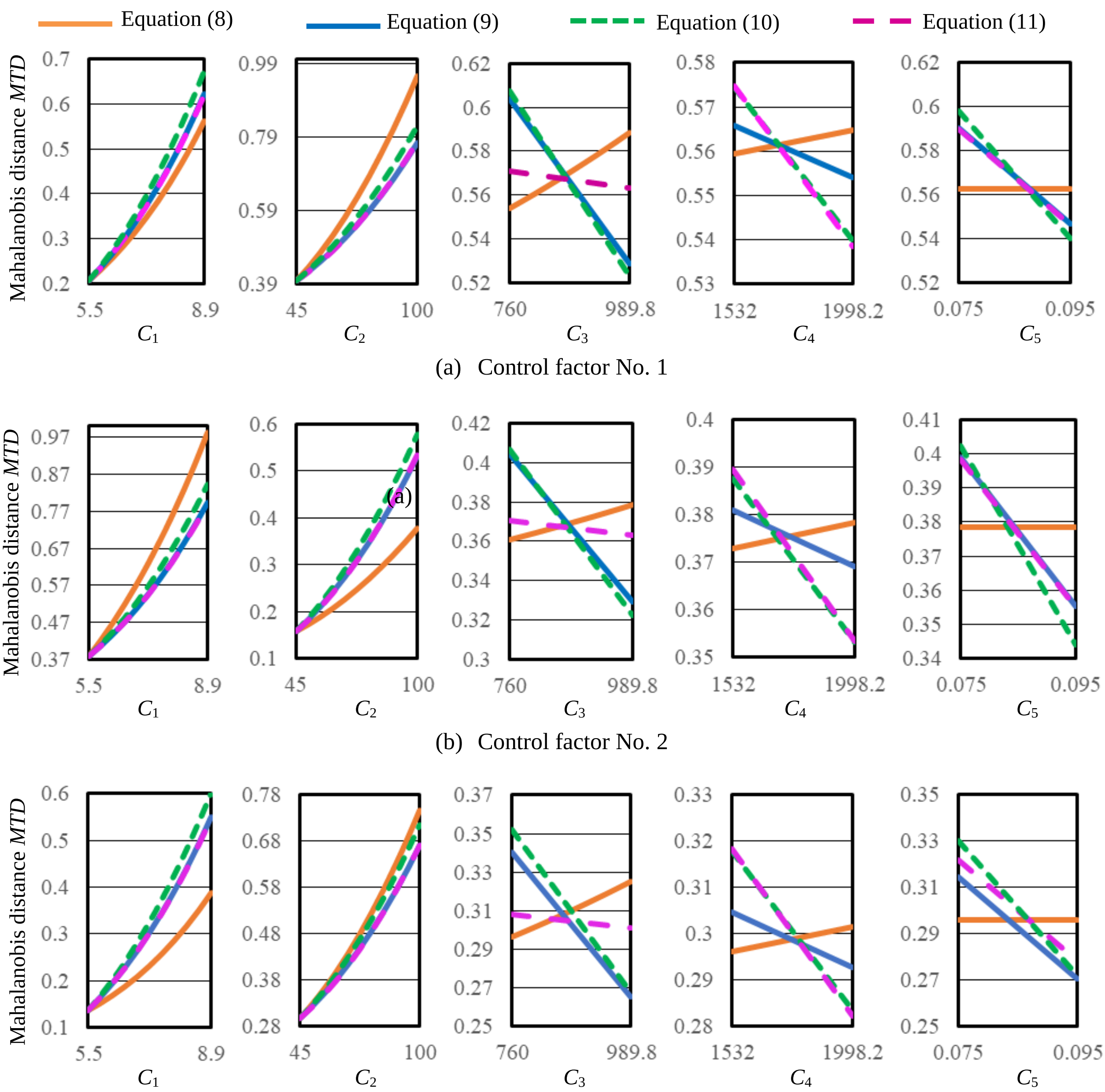
Table 7. Maximum differences between $MTD_{eq(8)}$ and $MTD_{eq(9)}$ in Fig. 3

$MTD_{eq\,(8)}$ and $MTD_{eq\,(9)}$		Control factors				
		C_1	C_2	C_3	C_4	C_5
Mahalanobis distance	No. 1	0.06	0.18	0.06	0.007	0.028
	No. 2	0.19	0.16	0.049	0.008	0.021
	No. 3	0.16	0.08	0.075	0.013	0.025
	Average	0.14	0.14	0.061	0.009	0.025
	Maximum	0.19	0.18	0.075	0.013	0.028
	Maximum	0.19	0.18	0.075	0.013	0.028

*Interaction : C_1 and C_2, Small interaction: C_3 (See Table 6)

4.2. ANALYSIS OF THE NUMBER OF LEVEL VALUES' EFFECT ON THE ALGEBRAIC EXPRESSION USING THE OPTIMAL CONDITIONS IDENTIFICATION PROGRAM

The relationship between the number of level values in the algebraic expression and the calculation accuracy of algebraic expressions are studied using equation (8) and the data of Table 6. Under the same conditions used in the previous section, for the five control factors, the minimum and maximum values of the control factors C_1, C_2, C_3, C_4, and C_5 are divided into two or four equal parts, and each level value is then divided into three parts. $MTD_{eq(10)}$ and $MTD_{eq(11)}$ are presented by algebraic expressions using the orthogonal arrays L16 and L25 respectively.



(a) Control factor No. 1

(b) Control factor No. 2

(c) Control factor No. 3

Fig. 4. Comparison of $MTD_{eq(8)}$, $MTD_{eq(9)}$, $MTD_{eq(10)}$ and $MTD_{eq(11)}$

$$MTD_{eq(10)} = 1.491542984\times10^{-2}\,C_1^2 - 7.793915299\times10^{-2}C_1 + 5.09466027\times10^{-5}C_2^2 + 2.634435622\times10^{-4}C_2 - 3.672685477\times10^{-4}C_3 - 7.432278636\times10^{-5}C_4 - 2.904576412C_5 + 0.614766 \qquad (10)$$

$$MTD_{eq(11)} = 7.62352914\times10^{-4}C_1^3 - 2.612521508\times10^{-3}C_1^2 + 3.830074326\times10^{-2}C_1 - 3.347276133\times10^{-9}C_2^3 + 5.119948222\times10^{-5}C_2^2 - 5.121389742\times10^{-4}C_2 - 3.181254134\times10^{-5}C_3 - 7.75142852\times10^{-5}C_4 - 2.175071456C_5 + 0.08795 \qquad (11)$$

The vertical axis was locally expanded to show the effect of the number of levels in Fig. 4, When the level number increases, the calculation accuracy of *MTD* by the algebraic expression models also increases. Thus, the proposed method becomes more accurate.

Table 8 shows the maximum difference of *MTD* between the mathematical model in Fig. 4 and the algebraic expressions of the Equations (9), (10) and (11). The results show that there is no significant difference in the $MTD_{eq\,(9)}$, $MTD_{eq\,(10)}$, and $MTD_{eq\,(11)}$ between C_1, C_2, and C_3. For C_4 and C_5 without interaction, the accuracy was partially improved by increasing the number of level, but the overall accuracy improvement is still not confirmed. Also, the local extension of the vertical axis revealed the effect of the number of levels, but it didn't show a big difference between $MTD_{eq\,(9)}$, $MTD_{eq\,(10)}$, and $MTD_{eq\,(11)}$ so it can be assumed that there is no interaction. Therefore, the calculation's accuracy is not significantly affected by the fluctuations of the number of levels 3, 4, and 5.

Table 8. Maximum differences between MTD_{eq} (8) and MTD_{eq} (9) in Fig. 3

At making equation (9), (level number＋1) ↴			Control factors				
			C_1	C_2	C_3	C_4	C_5
MTD	No. 1	3	0.11	0.14	0.065	0.025	0.036
		4	0.06	0.18	0.06	0.007	0.028
		5	0.058	0.18	0.025	0.026	0.027
	No. 2	3	0.14	0.2	0.056	0.0251	0.034
		4	0.191	0.155	0.049	0.008	0.021
		5	0.193	0.156	0.015	0.025	0.023
	No. 3	3	0.21	0.03	0.058	0.0219	0.034
		4	0.16	0.08	0.075	0.013	0.025
		5	0.16	0.076	0.024	0.0222	0.026
	Average	3	0.15	0.12	0.06	0.024	0.035
		4	0.1366	0.14	0.061	0.009	0.025
		5	0.137	0.137	0.02	0.0244	0.0253
	Maximum	3	0.21	0.2	0.065	0.0251	0.036
		4	0.19	0.18	0.075	0.013	0.028
		5	0.193	0.18	0.025	0.026	0.027

4.3. CONSIDERATIONS FOR THE EFFECTIVE USE OF THE PROPOSED METHOD

The interaction of control factors often results in extremely complex non-linear characteristics of the MTD. This is one of the reasons why MTS is used in AI. However, when MTS is used for anything other than fully automated AI control, the several interactions between the control factors in the MTS make highly accurate control difficult. However, on the other hand, the interaction of these factors can be clarified, by proactively incorporating interactions for manufacturing, controlling or management, it is possible to achieve high-precision control, improved production and quicker and more effective control and management. As a requirement for optimal control or management using MTS, it is important to understand (1) the presence or absence of an interaction, (2) the magnitude of the interaction, and (3) the conditions under which the interaction occurs. The proposed method can clarify these requirements and is considered to be very effective in industry.

My future work is to verify the scientific methodology, and to evaluate the proposed method using actual measured data of friction coefficients between machine parts, then I will report in the next paper.

5. CONCLUSION

In this study, an approach identifying and analyzing the interactions between the control factors in an MTS is developed. The main results of this research are the following:

1. A method for exploring the interaction between the control factors in MTS using an optimal program was proposed and the algorithm for this was explained.
2. The proposed method allows to determine the presence or absence of interactions, identify the control factors involved in the interactions and understand the effects of the interactions.
3. The calculation accuracy and error factors of the proposed method were discussed.
4. The effectiveness of the proposed method and the effective use of MTS were discussed. The results showed that the proposed method is effective for high-precision control, high productivity, quicker manufacturing and effective management in industry.

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