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*multi-objective optimization,*
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# USING TOPSIS METHOD FOR SOLVING MULTI-OBJECTIVE OPTIMIZATION OF A TWO-STAGE HELICAL GEARBOX WITH FIRST STAGE DOUBLE GEAR-SETS

In order to build a two-stage helical gearbox (THG) with first stage double gear-sets (FSDG), the multi-criteria decision-making (MCDM) method is introduced in this research as a new approach to solving the multi-objective optimization problem (MOOP). The study's objective is to determine the best primary design factors that will increase gearbox efficiency and decrease gearbox mass. To that end, the first stage's gear ratio and the first and second stages' coefficients of wheel face width (CWFW) were chosen as the three main design elements. Furthermore, two distinct goals were analyzed: the lowest gearbox mass and the highest gearbox efficiency. Additionally, the MOOP is carried out in two steps: phase 1 solves the single-objective optimization problem to close the gap between variable levels, and phase 2 solves the MOOP to determine the optimal primary design factors. Furthermore, the TOPSIS approach was selected to address the MOOP problem. For the first time, an MCDM technique is used to solve the MOOP of a helical gearbox with FSDG and the power losses during idle motion in order to determine gearbox efficiency was taken into the investigation for the gearbox. When designing the gearbox, the optimal values for three crucial design parameters were ascertained using the study's results.

## 1. INTRODUCTION

Helical gearboxes are extensively utilized in a wide range of industrial applications due to their inexpensive, dependable operation, and straightforward design. Designing a high-efficiency gearbox to reduce power loss and conserve energy is therefore one of the key needs. In addition, a gearbox must guarantee that other parameters like length, mass, or volume are minimal. As a result, it is required to solve a multi-objective optimization problem with a high gearbox efficiency requirement and other requirements.

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Numerous studies on multi-objective optimization of helical gearboxes, including the maximum gearbox efficiency function, have been conducted up to this point. In order to minimize the transmission volume and power losses, D. Miller et al. [1] carried out a multi-objective optimization of gear pair parameters using the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) technique. The gear module, face width, pinion and wheel profile shift coefficients, and pinion tooth count were the optimization variables used in this work. Additionally, the impact of the friction coefficient, sliding velocities, and nor-mal load on the gearing efficiency were examined. The study concluded that a trade-off between efficiency and volume is necessary, and that a lower gear module, a lower face width, higher profile shift coefficients, and a higher pinion tooth count produce satisfactory results for both goals. Maruti Patil et al. [2] also optimized a THG using the NSGA-II approach. Two goal functions were used in this study: the lowest gearbox volume and the lowest gearbox power loss overall. Numerous limitations were also considered, including tribological limitations, pitting stress, and bending stress. It was found that the multi-objective approach reduces the gearbox's overall power loss by half and that solutions derived from single objective minimization without tribological constraints had a significant probability of wear failure. It was also demonstrated that multi-objective optimization produced smaller power losses in comparison to single-objective optimization under tribological constraints. In order to reduce power loss and vibrational excitation caused by meshing, Emna B.Y. et al. [3] carried out a multi-objective optimization study of a gear unit utilizing the NSGA-II method in a multi-scale approach that goes from the gear con-tact to the entire transmission. Based on the results, it can be concluded that using both macro and micro geometry parameters simultaneously during multi-objective optimization yields different results than using macro geometry parameters first and micro geometry parameters second. Additionally, a comparison is done between the total power loss in the single stage gear unit and the local power loss caused by gear tooth friction in terms of design variable values in order to investigate the significance of considering the entire gear unit.

A helical gear pair's macro shape was optimized in [4] for low weight, high efficiency, and low noise. Trends of the best solutions for five combinations of the three goals were also examined. The study's goals in this work were the gear mass, gear efficiency, and transmission error. The objectives were scored and standardized in order to examine the outcomes. It was observed that, when mass, efficiency, and transmission error were taken into account, the majority of the top rankings were from the best options. For low weight, high efficiency, and low noise, the gear optimization process should take these three goals into account. Using the NSGA-II approach, Emund S.M. and Rajesh A. optimized a two-stage spur gearbox [5]. Three goals are simultaneously studied in this work: volume, power output, and center distance. Three design constraints and eight design variables were also chosen. The study's findings showed that, in comparison to power output and center distance, the variables related to the module, pinion tooth number, and face-width had a greater influence on volume.

A study to jointly optimize a gearbox and an electric motor for the purpose of de-signing an electric vehicle drive system was presented in [6]. The work's goal functions are to minimize the drive system's weight and overall energy loss. The optimization outcomes are contrasted with earlier findings to highlight collaborative optimization's further potential. It has been observed that when the drive system is optimized overall, raising the gear ratio raises the system's overall efficiency. In order to optimize a three-stage wind turbine gearbox, A. Kumar et al. [7] obtained consideration two goal functions: minimizing weight and minimizing power loss while taking into account the standard mechanical design restrictions as well as tribological constraints. In addition, three distinct gear tooth involute profiles - unmodified, smooth meshing, and high load capacity - are taken into account. At the recommended speed of 20 (rpm), these three profiles are evaluated using various synthetic-based ISO VG PAO (Polyalphaolefin) oils. Using ISO VG PAO 320, 680, and 1000 oils, the gearbox's results are compared with and without tribological limitation. PAO 320 oil performs better than the other two grades (PAO 680 and 1000), according to the results. Additionally, power loss is significantly decreased with tribological restriction for the selected model when comparing it with and without. A multi-objective optimization study of a two-stage spur gearbox under a wide range of constraints was carried out by M. Partil et al. [8]. Minimum volume and minimum gearbox power losses are the study's goals. The findings suggest that solutions derived from single objective minimization have a significant likelihood of experiencing wear failure. Additionally, when utilizing multi-goal optimization as opposed to single objective optimization, the overall power loss is cut in half.

Grey relation analysis (GRA) and the Taguchi technique were recently used by X.H. Le and N.P. Vu [9] to investigate the multi-objective optimization problem of building a two-stage helical gearbox. The aim of this study is to determine the ideal fundamental de-sign parameters that enhance gearbox efficiency while decreasing gearbox mass. In order to identify the optimal key design elements for a two-stage helical gearbox, a multi-target optimization problem was solved using the Taguchi and GRA methods in [10]. Two goals were examined in this work: the lowest possible gearbox height and the highest possible gearbox efficiency. Moreover, similar methods were used to optimize a THG with second stage double gear-sets in [11] in order to increase efficiency and reduce gearbox mass.

While numerous multi-objective optimizations have been performed to increase gearbox efficiency, the impact of a gearbox's primary design factors on efficiency has not been studied. Furthermore, no study has yet been conducted to tackle the multi-objective optimization problem utilizing the MCDM technique. This work used the MCDM method to perform multi-objective optimization research for a two-stage helical gearbox. Additionally, two different objectives were looked into: reducing gearbox mass and raising gearbox efficiency. This paper looked at three optimal primary design parameters for the two-stage helical gearbox. Among these are the first stage's gear ratio and the combined weight for both stages. Furthermore, the optimization task was approached using the TOP-SIS method, and the weights of the criteria were determined using the Entropy method. One of the main conclusions of the research is the suggestion to apply an MCDM technique to solve multi-objective optimization problems in conjunction with two-step problem solving, tackling single- and multi-objective problems. Moreover, the problem's solutions are more effective than those of earlier studies.

## 2. OPTIMIZATION PROBLEM

In this part, the gearbox mass and efficiency are first calculated in order to build the optimization problem. Next, the specified objective functions and constraints are given. To facilitate calculations, the nomenclatures used in the optimization problem are presented in Table 1.

Table 1. The nomenclature

| PARAMETER | NOMENCLATURE | UNITS |
|---|---|---|
| Arc of approach on i stage | $\beta_{ai}$ | |
| Allowable shear stress of shaft material | $[\tau]$ | MPa |
| Allowable contact stress of stages i (i=1÷2) | $AS_i$ | Mpa |
| Arc of recess on i stage | $\beta_{ri}$ | |
| Base-circle radius of the pinion | $R_{01i}$ | mm |
| Base-circle radius of the gear | $R_{02i}$ | mm |
| Center distance of stage 1 | $a_{w1}$ | mm |
| Center distance of stage 2 | $a_{w2}$ | mm |
| Contacting load ratio for pitting resistance | $k_{H\beta}$ | - |
| Diameter of shaft i | $d_{si}$ | mm |
| Efficiency of a helical gearboxt | $\eta_{hb}$ | - |
| Efficieny of the i stage of the gearbox | $\eta_{gi}$ | - |
| Efficiency of a helical gear unit | $\eta_{hg}$ | - |
| Efficiency of a rolling bearing pair | $\eta_b$ | - |
| Friction coefficient | f | - |
| Friction coefficient of bearing | $f_b$ | - |
| Gearbox mass | $m_{gb}$ | kg |
| Gear mass | $m_g$ | kg |
| Gearbox housing mass | $m_{gh}$ | kg |
| Gear mass of the first stage | $m_{g1}$ | kg |
| Gear mass of the second stage | $m_{g2}$ | kg |
| Gear ratio of stage 1 | $u_1$ | - |
| Gear ratio of stage 2 | $u_2$ | - |
| Gearbox ratio | $u_{gb}$ | - |
| Gear width of stage 1 | $b_{w1}$ | mm |
| Gear width of stage 2 | $b_{w2}$ | mm |
| Gearbox housing volume | $V_{gh}$ | $dm^3$ |
| Hydraulic moment of power losses | $T_H$ | Nm |
| ISO Viscosity Grades number. | $VG_{40}$ | |
| Load of bearing i | Fi | N |
| Length of shaft i | $l_{si}$ | mm |
| Mass density of gearbox housing materials | $\rho_{gh}$ | $kg/m^3$ |
| Material coefficient | $k_a$ | $Mpa^{1/3}$ |
| Mass of shaft j (j=1÷3) | $m_{sj}$ | kg |
| Mass density of shaft material | $\rho_s$ | $kg/m^3$ |
| Output torque | $T_{out}$ | Nmm |
| Outside radius of the pinion | $R_{e1i}$ | mm |
| Outside radius of the gear | $R_{e2i}$ | mm |
| Pitch diameter of the pinion of stage 1 | $d_{w11}$ | mm |
| Pitch diameter of the gear of stage 2 | $d_{w21}$ | mm |
| Pitch diameter of the pinion of stage 2 | $d_{w12}$ | mm |
| Pitch diameter of the gear of stage 2 | $d_{w22}$ | mm |
| Power loss in the gears | Plg | Kw |
| Power loss in the bearings | Plb | Kw |
| Power loss in the seals | Pls | Kw |
| Power loss in the idle motion | Pzo | Kw |
| Pressure angle | $\alpha$ | rad. |
| Peripheral speed of bearing | $v_b$ | m/s |
| Shaft mass | $m_s$ | kg |
| Sliding velocity of gear | v | m/s |
| Total power loss in the gearbox | Pl | |
| Torque on the pinion of stage i (i=1÷2) | $T_{1i}$ | Nmm |
| Volume coefficients of the pinion | $e_1$ | - |
| Volume coefficients of the gear | $e_2$ | - |
| Volumes of bottom housing A | $V_A$ | $dm^3$ |
| Volumes of bottom housing B | $V_B$ | $dm^3$ |
| Volumes of bottom housing B | $V_C$ | $dm^3$ |
| Wheel face width coefficient of stage 1 | $X_{ba1}$ | - |
| Wheel face width coefficient of stage 2 | $X_{ba2}$ | - |
| Weight density of gear materials | $\rho_g$ | $kg/m^3$ |

## 2.1. FINDING GEARBOX MASS

The gearbox mass $m_{gb}$ is calculated by:

$$m_{gb} = m_{gh} + m_g + m_s \tag{1}$$

In which, $m_{gh}$, $m_g$, and $m_s$ can be determined in detail as follows:

+) Finding $m_{gh}$:

In this work, $m_{gh}$ is found by:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \tag{2}$$

With $V_{gh}$ is calculated by (Fig. 1):

$$V_{gh} = 2 \cdot V_A + 2 \cdot V_B + 2 \cdot V_C \tag{3}$$

where:

$$V_A = L \cdot H \cdot S_G \tag{4}$$

$$V_B = L \cdot B_1 \cdot 1.5 \cdot S_G \tag{5}$$

$$V_C = B_2 \cdot H \cdot S_G = (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \tag{6}$$

In the above Equations, $L$, $H$, $B_1$, and $S_G$ can be found by:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22}/2 + 22.5)/0.975 \text{ [12]} \tag{7}$$

$$H = max(d_{w21}; d_{w22}) + 8.5 \cdot S_G \tag{8}$$

$$B_1 = b_{w1} + b_{w2} + 6 \cdot S_G \tag{9}$$

$$S_G = 0.005 \cdot L + 4.5 \text{ [12]} \tag{10}$$

+) Finding $m_g$:

$$m_g = 2 \cdot m_{g1} + m_{g2} \tag{11}$$

In (11):

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4}\right) \tag{12}$$

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4}\right) \tag{13}$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \tag{14}$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \tag{15}$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{16}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{17}$$

In Equations (12) to (17), $i$=1÷2; $\rho_g = 7800$ $(kg/m^3)$ as the gear material is steel; $e_1 = 1$ and $e_2 = 0.6$ [12]; and $a_{wi}$ is determined by [12]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{18}$$

where: $T_{1i}$ ($i$=1÷2) can be found by:

$$T_{11} = T_{out}/\left(2 \cdot u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \tag{19}$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \tag{20}$$

+) Finding $m_s$:

In this study, $m_s$ can be found by:

$$m_s = \sum_{j=1}^{3} m_{sj} \tag{21}$$

In which

$$m_{sj} = \rho_s \cdot \pi \cdot d_{sj}^2 \cdot l_{sj}/4 \tag{22}$$

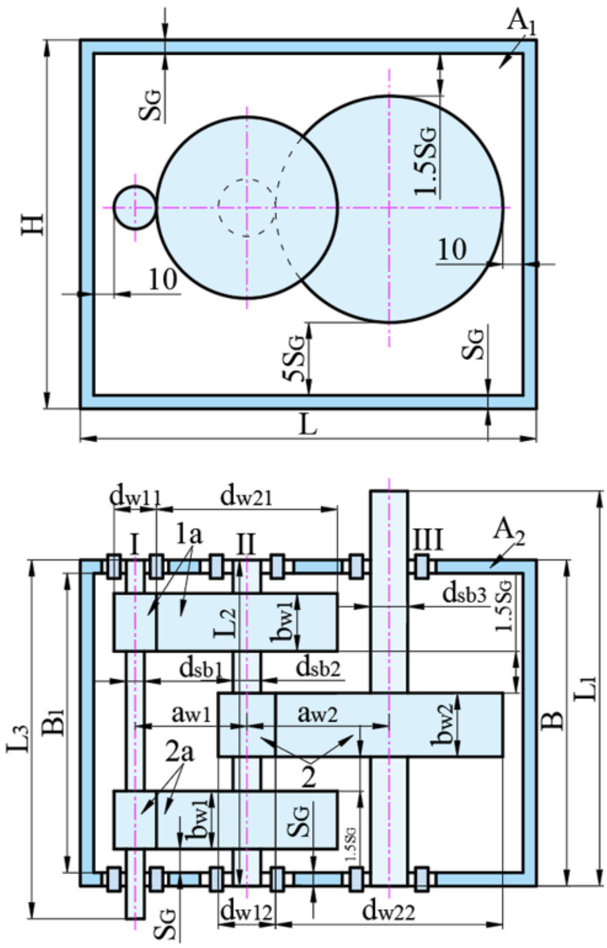



Fig. 1. Calculated schema

In (22), $l_{sj}$ can be found by (see Fig. 1):

$$l_{s1} = B_1 + 1.2 \cdot d_{s1} \tag{23}$$

$$l_{s2} = B_1 \tag{24}$$

$$l_{s3} = B_1 + 1.2 \cdot d_{s3} \tag{25}$$

In (22), $d_{sj}$ ($j$=1÷3) is determined by [12]:

$$d_{sj} = [T_{1j}/(0.2 \cdot [\tau])]^{1/3} \tag{26}$$

In the above Equations, $\rho_g = 7800\ (kg/m^3)$ and $\rho_s = 7800\ (kg/m^3)$ because steel was chosen as the material of the gears and the shafts; $[\tau]$= 17 (Mpa) [12].

## 2.2. FINDING GEARBOX EFFICIENCY

The efficiency of the gearbox (%) is found by:

$$\eta_{gb} = 100\text{-}\frac{100 \cdot P_l}{P_{in}} \tag{27}$$

In which, $P_l$ is calculated by [12],

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{28}$$

where: $P_{lg}$, $P_{lb}$, $P_{ls}$, and $P_{zo}$ can be determined by:
+) Finding $P_{lg}$:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \tag{29}$$

In which,

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right) \tag{30}$$

Where: $\eta_{gi}$ is calculated by [12]:

$$\eta_{gi} = 1\text{-}\left(\frac{1+1/u_i}{\beta_{ai}+\beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot (\beta_{ai}^2 + \beta_{ri}^2) \tag{31}$$

In which, $\beta_{ai}$ and $\beta_{ri}$ can be found by [12]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2\text{-}R_{02i}^2\right)^{1/2}\text{-}R_{2i}\cdot\sin\alpha}{R_{01i}} \tag{32}$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2\text{-}R_{01i}^2\right)^{1/2}\text{-}R_{1i}\cdot\sin\alpha}{R_{01i}} \tag{33}$$

Where $f$ is found by [10]:
- If $v \leq 0.424$ (m/s):

$$f = -0.0877 \cdot v + 0.0525 \tag{34}$$

- If $v > 0.424$ (m/s):

$$f = 0.0028 \cdot v + 0.0104 \tag{35}$$

+) Calculation of $P_{lb}$ [12]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \tag{36}$$

In which, $i$=1÷6 and $f_b = 0.0011$ because the radical ball bearings with angular contact were selected [13].
+) Finding $P_s$ [12]:

$$P_s = \sum_{i=1}^{2} P_{si} \tag{37}$$

Wherein, $i$ is the ordinal number of seal ($i$=1÷2); and $P_{si}$ is determined by:

$$P_{si} = [145\text{-}1.6 \cdot t_{oil} + 350 \cdot \log\log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \tag{38}$$

+) Finding $P_{zo}$ [12]:

$$P_{Z0} = \sum_{i=1}^{k} T_{Hi} \cdot \frac{\pi \cdot n_i}{30} \tag{39}$$

In (39), $k$ is the gear pair number ($k$=2); $n$ is the revolution number of driven gear; $T_{Hi}$ can be found by [12]:

$$T_{Hi} = C_{Sp} \cdot C_1 \cdot e^{\frac{C_2 \cdot v}{v_{t0}}} \tag{40}$$

Where, $C_{Sp} = 1$ for stage 1 when the involved oil has to pass until the mesh (Fig. 2) and in case of stage 2, $C_{Sp}$ can be found by (Fig. 2):

$$C_{Sp} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_C}\right)^{1.5} \cdot \frac{2 \cdot h_C}{l_{hi}} \tag{41}$$

In which, $l_{hi}$ is determined by [13]:

$$l_{hi} = (1.2 \div 2.0) \cdot d_{a2i} \tag{42}$$

In (40), $C_1$ and $C_2$ can be found by [13]:

$$C_1 = 0.063 \cdot \left(\frac{e_1+e_2}{e_0}\right) + 0.0128 \cdot \left(\frac{b}{b_0}\right) \tag{43}$$

$$C_2 = \frac{e_1+e_2}{80 \cdot e_0} + 0.2 \tag{44}$$

Wherein $e_0 = b_0$ =10 (mm).

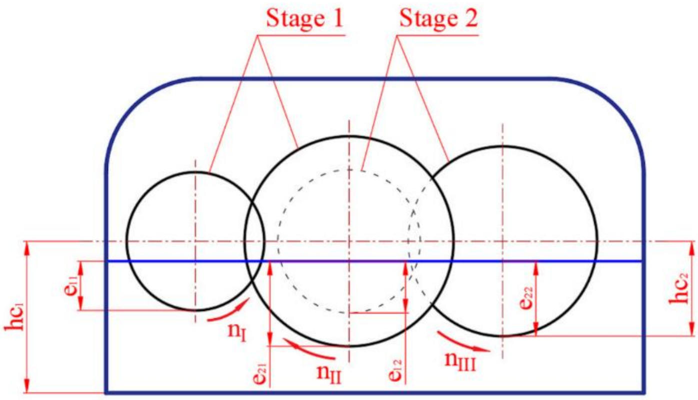


Fig. 2. Calculated schema for bath lubrication factors

### 2.3. OBJECTIVE FUNCTIONS AND CONSTRAINS

#### 2.3.1. OBJECTIVES FUNCTIONS

In this study, the multi-objective optimization problem includes two single objectives:
– Minimizing the mass of the gearbox:

$$\min f_1(X) = m_{gb} \tag{45}$$

– Maximizing the efficiency of the gearbox:

$$\min f_2(X) = \eta_{gb} \tag{46}$$

In which, $X$ is the design variable vector. In this work, three main design parameters including $u_1, Xba_1, Xba_2$ were chosen as variables for the optimization problem and we have:

$$X = \{u_1, Xba_1, Xba_2\} \tag{47}$$

#### 2.3.2. CONSTRAINS

The constraints that follow must be met by the multi-objective function:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \tag{48}$$

$$0.25 \le Xba_1 \le 0.4 \text{ and } 0.25 \le Xba_2 \le 0.4 \tag{49}$$

## 3. METHODOLOGY

### 3.1. METHOD FOR SOLVING MULTI-OBJECTIVE OPTIMIZATION

Three primary design factors are chosen as variables for the multi-objective optimization problem, as mentioned in section 2.3. Table 2 lists these variables along with their minimum and maximum values. In order to determine the ideal values for the three primary design variables, the multi-objective optimization problem with two objectives: minimum gearbox mass and maximum gearbox efficiency-was addressed in this work using the TOPSIS approach.

Table 2. Input parameters

| PARAMETER | SYMBOL | LOWER LIMIT | UPPER LIMIT |
|---|---|---|---|
| Gearbox ratio of stage 1 | $u_1$ | 1 | 9 |
| CWFW of stage 1 | *Xba1* | 0.25 | 0.4 |
| CWFW of stage 2 | *Xba2* | 0.25 | 0.4 |

To solve the multi-objective optimization problem for a THG with FSDG, a simulation experiment was built. Additionally, because this is a simulation experiment, there is no restriction on the number of experiments conducted thanks to the full factorial design. Because there are three experimental variables (as previously specified) and five levels for each variable, the total number of experiments will be $5^3 = 125$. However, Table 2 indicates that *u1* has the broadest spread among the three specified variables (ranging from 1 to 9). As a result, even with five levels, there was still a significant gap between the levels of this variable (in this case, it is ((9–1)/4=2). An approach to addressing multi-objective issues was proposed in an attempt to reduce this discrepancy, expedite the process, and boost accuracy.
Figure 3 describes the process diagram used to solve the multi-objective problem.

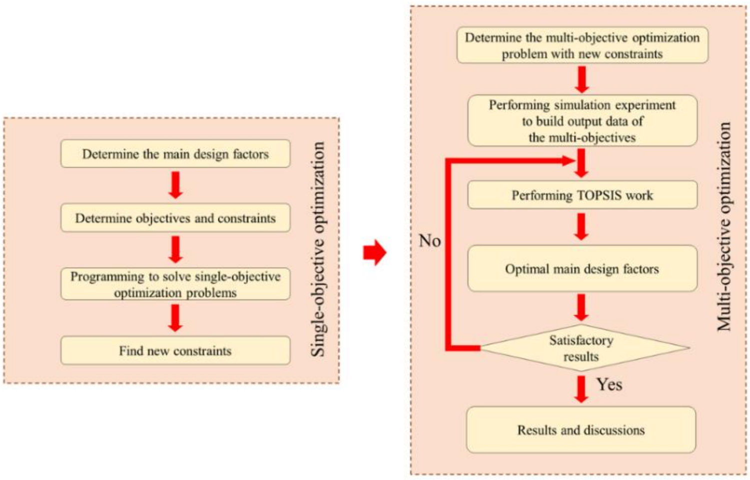



Fig. 3. The process flow chart for solving multi-objective problem

This procedure is broken down into two phases: phase 1 factors reduce the gap between levels by solving the single-objective optimization problem, and phase 2 factors find the optimal primary design by solving the multi-objective optimization problem. Additionally, in the process of solving the multi-objective problem, the TOPSIS issue will be rerun using the smaller distance between two levels of the $u_1$ if the variable's levels are not sufficiently close to one another or if the best answer is not appropriate for the requirement (Fig. 3).

### 3.2. METHOD FOR SOLVING MCDM PROBLEM

To apply the TOPSIS approach, the following procedures must be completed [16]: Creating initial decision-making matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \tag{50}$$

With *n* and *m* are the criterion and alternative numbers.

– Determining normalized values $k_{ij}$ by:

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{51}$$

– Calculating weighted normalized decision matrix by:

$$l_{ij} = w_j \times k_{ij} \tag{52}$$

– Finding the best alternative $A^+$ and the worst s alternative $A^-$ by:

$$A^+ = \{l_1^+, l_2^+, \ldots, l_j^+, \ldots, l_n^+\} \tag{53}$$

$$A^- = \{l_1^-, l_2^-, \ldots, l_j^-, \ldots, l_n^-\} \tag{54}$$

In which, $l_j^+$ and $l_j^-$ are the best and worst values of criterion *j* (*j*=1,2, ..., n).

– Finding better options $D_i^+$ and worse options $D_i^-$ by:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(l_{ij} - l_j^+\right)^2} \tag{55}$$

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(l_{ij} - l_j^+\right)^2} \tag{56}$$

– Determining closeness coefficient $R_i$ of each alternative by:

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{57}$$

– Ranking options by maximizing the value of *R*.

### 3.3. METHOD FOR FINDING THE WEIGHT OF CRITERIA

In this work, the Entropy technique was used to establish the weights of the criteria. The steps listed below can be utilized for putting this method into practice [17].

– Finding indicator normalized values:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} \tag{58}$$

– Calculating the Entropy for each indicator:

$$me_j = -\sum_{i=1}^{m} \left[p_{ij} \times \ln(p_{ij})\right] - \left(1 - \sum_{i=1}^{m} p_{ij}\right) \times \ln\left(1 - \sum_{i=1}^{m} p_{ij}\right) \tag{59}$$

– Determining the weight of each indicator:

$$w_j = \frac{1 - me_j}{\sum_{j=1}^{m} (1 - me_j)} \tag{60}$$

## 4. SINGLE-OBJECTIVE OPTIMIZATION

In this study, the direct search strategy is used to solve the single-objective optimization problem. Furthermore, a computer program has been created to solve two single-objective problems: reducing gearbox mass and enhancing gearbox efficiency. The following are some of the program's results' figures and observations: Fig. 4 shows the relationship between $\eta_{gb}$ and $u_1$. It was discovered that $\eta_{gb}$ reaches its maximum at an optimal value of $u_1$. Additionally, Fig. 5 shows the connection between $u_1$ and $m_{gb}$. When $u_1$ is at its optimal value, $m_{gb}$ reaches its lowest value (Fig. 5). Figure 6 and Fig. 7 show the relationship between $X_{ba1}$ and $X_{ba2}$ with mgb and $\eta_{gb}$, respectively. These results (Fig. 6.a and Fig. 7.a) demonstrate that $m_{gb}$ will rise in response to increases in $X_{ba1}$ and $X_{ba2}$. Conversely, as $X_{ba1}$ and $X_{ba2}$ increase, $\eta_{gb}$ decreases (Fig. 6.b and Fig. 7.b). Figure 8 illustrates the link between the ideal gear ratio, $u_1$, for the first stage and the total gearbox ratio, $u_t$. Additionally, Table 3 displays newly derived constraints for the variable $u_1$.

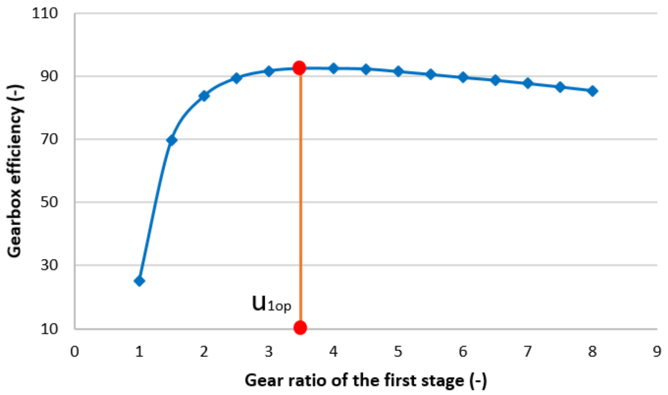



Fig. 4. Relation between gearbox efficiency and first stage gear ratio

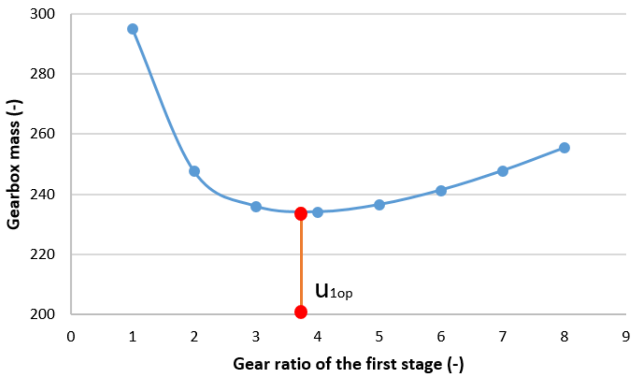



Fig. 5. Relation between gearbox mass and first stage gear ratio

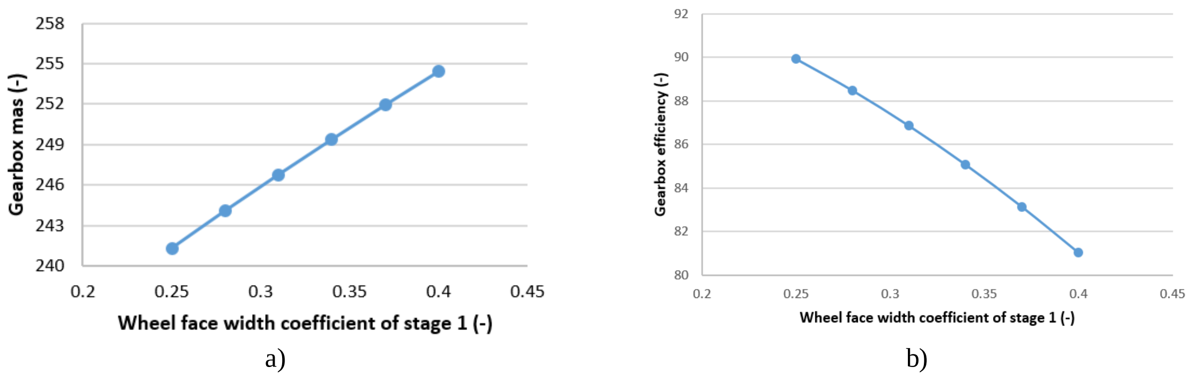
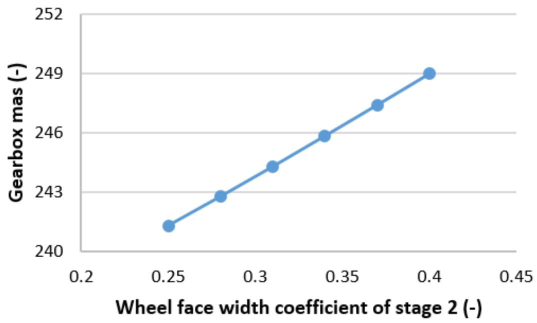
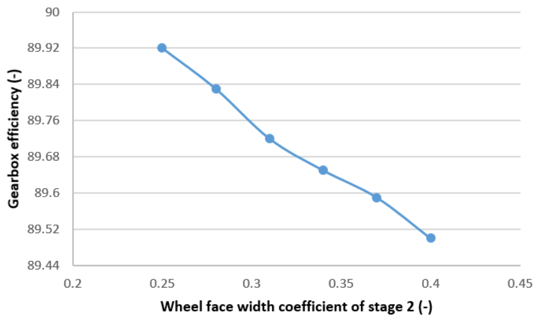



Fig. 6. Relation between $X_{ba1}$ and gearbox mass (a) and gearbox efficiency (b)




a)




b)

Fig. 7. Relation between $X_{ba2}$ and gearbox mass (a) and gearbox efficiency (b)

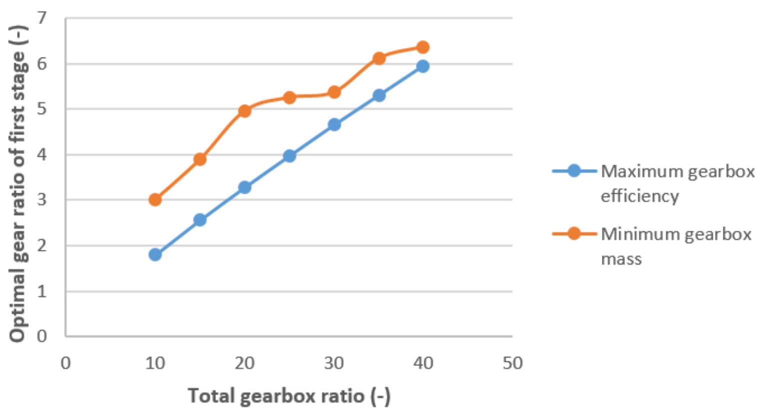



Fig. 8. Optimum gear ratio of stage 1 versus total gearbox ratio

Table 3. New constraints of $u_1$

| $u_t$ | $u_1$ | |
|---|---|---|
| | LOWER LIMIT | UPPER LIMIT |
| 10 | 1.70 | 3.12 |
| 15 | 2.46 | 4.00 |
| 20 | 3.18 | 5.06 |
| 25 | 3.87 | 5.35 |
| 30 | 4.55 | 5.47 |
| 35 | 5.20 | 6.21 |
| 40 | 5.85 | 6.46 |

## 5. MULTI-OBJECTIVE OPTIMIZATION

A software application has been created to do simulation research. The gearbox ratios 10, 15, 20, 25, 30, 35, and 40 were all considered for the analysis. This problem with $u_{gb}$=10 has the answers displayed below. This total gearbox ratio was used for 125 initial testing cycles (as specified in section 3). The outcome values of the experiment, the gearbox mass and gearbox efficiency, will be used as input parameters by TOPSIS to solve the multi-objective optimization problem. This procedure will be repeated until there is less than 0.02 separating two levels of $u_1$. The primary design parameters and output responses for $u_{gb}$=10 in the fifth and final iteration of the TOPSIS experiment are shown in Table 4. The weights of the criteria have been established using the Entropy technique (see Section 3.3) as follows: First, use equation (58) to get the normalized values of $p_{ij}$. Equation (59) is used to determine each indicator $m_{ej}$'s entropy value. Finally, use Equation (60) to find the weight of the criteria $w_j$. The weights of $m_{gb}$ and $\eta_{gb}$ for the most recent TOPSIS work run were discovered to be 0.4877 and 0.5123, respectively. Instructions for using the TOPSIS technique in multi-objective decision making are given in Section 3.2. Consequently, the normalized values of $k_{ij}$ and the normalized weighted values of $l_{ij}$ are obtained by Equation (51) and Equation (52). For $m_{gb}$ and $\eta_{gb}$, the $A^+$ and $A^-$ values are obtained using equations (53) and (54) respectively. Besides, $D_i^+$ and $D_i^-$ values were calculated with the use of Formulas (56 and 55). Finally, the ratio $R_i$ was obtained by using Equation (57). Table 5 (for the final run of TOPSIS work) shows the outcomes of the option ranking and the computing of numerous parameters using the TOSIS approach. Out of all the possibilities given, option 26 is the most ideal one, according to the table. The optimal values for the main design elements are therefore $u_1$ = 2.485, $X_{ba1}$ = 0.25, and $X_{ba2}$ = 0.25 (see Table 4).

Table 4. Main design parameters and output results for $u_{gb}$=10 in the 5th run of TOPSIS

| TRIAL. | $u_1$ | $X_{ba1}$ | $X_{ba2}$ | $m_{gb}$ (kg) | $\eta_{gb}$(%) |
|---|---|---|---|---|---|
| 1 | 2.47 | 0.25 | 0.25 | 237.52 | 93.96 |
| 2 | 2.47 | 0.25 | 0.2875 | 239.36 | 94.18 |
| 3 | 2.47 | 0.25 | 0.325 | 241.29 | 94.21 |
| 4 | 2.47 | 0.25 | 0.3625 | 243.25 | 94.22 |
| 5 | 2.47 | 0.25 | 0.4 | 245.24 | 94.28 |
| 6 | 2.47 | 0.2875 | 0.25 | 240.39 | 93.22 |
| | ... | | | | |
| 25 | 2.47 | 0.4 | 0.4 | 256.03 | 90.64 |
| 26 | 2.485 | 0.25 | 0.25 | 237.56 | 94.17 |
| 27 | 2.485 | 0.25 | 0.2875 | 239.41 | 94.18 |
| | ... | | | | |
| 50 | 2.485 | 0.4 | 0.4 | 256.12 | 90.64 |
| 51 | 2.5 | 0.25 | 0.25 | 237.61 | 94.17 |
| 52 | 2.5 | 0.25 | 0.2875 | 239.46 | 94.18 |
| | ... | | | | |
| 74 | 2.5 | 0.4 | 0.3625 | 254.24 | 90.63 |
| 75 | 2.515 | 0.4 | 0.4 | 256.3 | 90.62 |
| 76 | 2.515 | 0.25 | 0.25 | 237.66 | 94.14 |
| | ... | | | | |
| 100 | 2.515 | 0.4 | 0.4 | 256.3 | 90.62 |
| 101 | 2.53 | 0.25 | 0.25 | 237.71 | 94.14 |
| 102 | 2.53 | 0.25 | 0.2875 | 239.56 | 94.15 |
| | ... | | | | |
| 123 | 2.53 | 0.4 | 0.325 | 252.48 | 90.63 |
| 124 | 2.53 | 0.4 | 0.3625 | 254.43 | 90.61 |
| 125 | 2.53 | 0.4 | 0.4 | 256.39 | 90.62 |

Table 5. Several calculated results and ranking of options by TOPSIS method for $u_{gb}$=10

| TRIAL. | $k_{ij}$ | | $l_{ij}$ | | $S_i^+$ | $S_i^-$ | $R_i$ | Rank |
|---|---|---|---|---|---|---|---|---|
| | $m_{gb}$ | $e_{gb}$ | $m_{gb}$ | $e_{gb}$ | | | | |
| 1 | 0.0860 | 0.0908 | 0.0419 | 0.0465 | 0.0002 | 0.0037 | 0.9593 | 5 |
| 2 | 0.0867 | 0.0910 | 0.0423 | 0.0466 | 0.0003 | 0.0035 | 0.9142 | 7 |
| 3 | 0.0874 | 0.0911 | 0.0426 | 0.0467 | 0.0007 | 0.0032 | 0.8286 | 22 |
| 4 | 0.0881 | 0.0911 | 0.0430 | 0.0467 | 0.0010 | 0.0029 | 0.7442 | 41 |
| 5 | 0.0888 | 0.0911 | 0.0433 | 0.0467 | 0.0014 | 0.0027 | 0.6642 | 68 |
| 6 | 0.0870 | 0.0901 | 0.0424 | 0.0462 | 0.0007 | 0.0031 | 0.8103 | 11 |
| | ... | | | | | | | |
| 25 | 0.0927 | 0.0876 | 0.0452 | 0.0449 | 0.0037 | 0.0001 | 0.0197 | 122 |
| 26 | 0.0860 | 0.0910 | 0.0419 | 0.0466 | 0.0001 | 0.0038 | 0.9857 | 1 |
| 27 | 0.0867 | 0.0910 | 0.0423 | 0.0466 | 0.0003 | 0.0035 | 0.9119 | 8 |
| | ... | | | | | | | |
| 50 | 0.0927 | 0.0876 | 0.0452 | 0.0449 | 0.0037 | 0.0001 | 0.0163 | 121 |
| 51 | 0.0860 | 0.0910 | 0.0420 | 0.0466 | 0.0001 | 0.0038 | 0.9852 | 2 |
| 52 | 0.0867 | 0.0910 | 0.0423 | 0.0466 | 0.0003 | 0.0035 | 0.9097 | 6 |
| | ... | | | | | | | |
| 74 | 0.0921 | 0.0876 | 0.0449 | 0.0449 | 0.0035 | 0.0004 | 0.0992 | 118 |
| 75 | 0.0928 | 0.0876 | 0.0453 | 0.0449 | 0.0038 | 0.0000 | 0.0088 | 124 |
| 76 | 0.0861 | 0.0910 | 0.0420 | 0.0466 | 0.0001 | 0.0038 | 0.9808 | 3 |
| | ... | | | | | | | |
| 100 | 0.0928 | 0.0876 | 0.0453 | 0.0449 | 0.0038 | 0.0000 | 0.0088 | 123 |
| 101 | 0.0861 | 0.0910 | 0.0420 | 0.0466 | 0.0001 | 0.0037 | 0.9798 | 4 |
| 102 | 0.0867 | 0.0910 | 0.0423 | 0.0466 | 0.0004 | 0.0035 | 0.9044 | 10 |
| | ... | | | | | | | |
| 123 | 0.0914 | 0.0876 | 0.0446 | 0.0449 | 0.0032 | 0.0007 | 0.1776 | 109 |
| 124 | 0.0921 | 0.0876 | 0.0449 | 0.0449 | 0.0035 | 0.0003 | 0.0903 | 120 |
| 125 | 0.0928 | 0.0876 | 0.0453 | 0.0449 | 0.0038 | 0.0000 | 0.0078 | 125 |

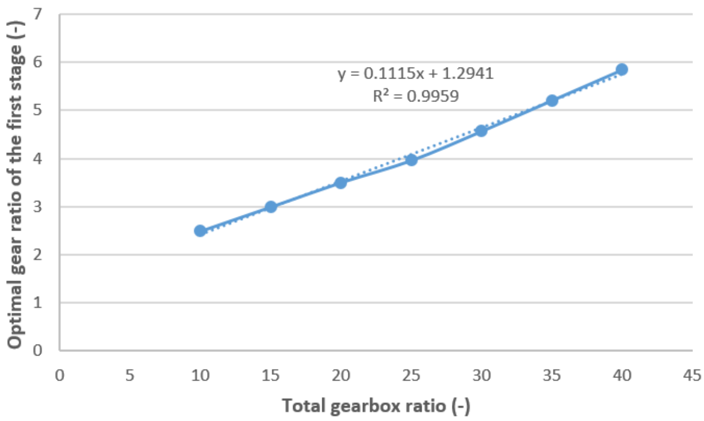



Fig. 9. Optimum gear ratio of stage 1 versus total gearbox ratio

Table 6 shows the optimal values for the main design parameters that correspond to the remaining $u_{gb}$ values of 10, 20, 25, 30, 35, and 40, continuing with the previous discussion. The following conclusions can be drawn from the data in Table 6:

Table 6. Optimum values of main design parameters

| NO. | $u_{gb}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| $u_1$ | 2.485 | 2.99 | 3.50 | 3.96 | 4.57 | 5.21 | 5.85 |
| $X_{ba1}$ | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| $X_{ba2}$ | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |

The smallest values for $X_{ba1}$ and $X_{ba2}$ that correspond to their optimal values are $X_{ba1}$=0.25 and $X_{ba2}$=0.25. This result follows well with the information that was stated in [10]. This is so that the necessary lowest gearbox mass may be obtained, and the coefficients $X_{ba1}$ and $X_{ba2}$ must be as small as possible. By decreasing these coefficients, the gear widths (represented by Equations (16) and (17) and, subsequently, the gear mass (represented by Equations (14) and (15)) can be reduced.

Figure 9 shows that the ideal values of $u_1$ and $u_{gb}$ clearly show a first-order relationship. Moreover, the optimal values of $u_1$ were found to be determined by the regression equation that follows (with $R^2$=0.9959):

$$u_1 = 0.1115 \cdot u_{gb} + 1.2941 \tag{61}$$

After determining $u_1$, the optimal value of $u_2$ can be found using the formula below:

$$u_2 = u_t/u_1 \tag{62}$$

## 6. CONCLUSION

The TOPSIS approach was used in this study to address the multi-objective optimization problem in the design of a THG with FSDG. The study's goal is to determine the most important design parameters that maximize gearbox efficiency while reducing gearbox mass. To do this, three key design elements were chosen: the CWFW for the first and second stages, and the first stage gear ratio. In addition, there are two steps in the multi-objective optimization problem solution process. The first step is centered on solving the single-objective optimization problem of reducing the difference between variable values, whereas the second step is concerned with determining the optimal primary de-sign factors. The following results were drawn from this work:

– The single-objective optimization problem improves up and simplifies the resolution of the multi-objective optimization problem by reducing the gap between variable levels.

– The three main design parameters for a THG, Equation (61) and Table 6, were recommended to have ideal values based on the study's findings.

– Two single objectives were assessed concerning the main design parameters.

– By repeatedly applying the TOPSIS technique until the desired results are attained ($u_1$ has an accuracy of less than 0.02), the multi-objective optimization problem can be solved more precisely.

– The experimental data' incredible degree of agreement with the proposed model of $u_1$ verifies their reliability.

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