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*NSGA-II, efficiency,
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Thanh Van DINH¹, Duong VU^{2*},
Pi Ngoc VU³, Binh Duc VU⁴,

A MULTI-OBJECTIVE OPTIMIZATION SOLUTION FOR TWO-STAGE HELICAL GEARBOX DESIGN WITH A NEW METHOD NSGA-II: COMPROMISING EFFICIENCY AND STRUCTURAL COMPACTNESS

The design of two-stage helical gearboxes inherently involves trade-offs between structural compactness and transmission efficiency. Traditional design methods often fail to capture these competing objectives simultaneously. This study presents a comprehensive multi-objective optimization approach using the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to identify optimal trade-offs between minimizing the cross-sectional area and maximizing the efficiency of a two-stage helical gearbox. Drawing from the strengths of evolutionary algorithms and integrating insights from prior literature on hybrid and decision-making-based optimization methods, the proposed model formulates the gearbox design problem with realistic constraints and evaluates the Pareto front of optimal solutions. The results demonstrate that NSGA-II provides a well-distributed set of non-dominated solutions, offering engineers greater flexibility in balancing performance and structural requirements. Comparative analysis with existing approaches highlights the effectiveness of the proposed method in simultaneously achieving compactness and energy efficiency in gearbox systems.

1. INTRODUCTION

Gearbox systems play a critical role in mechanical power transmission, particularly in applications requiring both compactness and efficiency such as automotive, wind energy, and industrial automation. Among various gearbox types, two-stage helical gearboxes are known for their ability to transmit high torque smoothly while maintaining relatively compact dimensions. However, the design of these systems typically involves complex trade-offs between structural parameters and performance indicators. Specifically, minimizing the

¹ Faculty of Mechanical Engineering, East Asia University of Technology, Vietnam

² School of Engineering Technology, Duy Tan University, Vietnam

³ Faculty of Mechanical Engineering, Thai Nguyen University of Technology, Vietnam

⁴ Faculty of Mechanical Engineering, Viet Tri University of Industry, Vietnam

* E-mail: duongvuastralia@gmail.com

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cross-sectional area (to save space and reduce material cost) often conflicts with maximizing transmission efficiency.

To address such design challenges, multi-objective optimization (MOO) techniques have become a powerful alternative to traditional deterministic or mono-objective design approaches. These methods enable simultaneous consideration of conflicting objectives and offer a range of Pareto-optimal solutions, from which engineers can choose the most suitable compromise.

1.1. EVOLUTIONARY ALGORITHMS FOR MULTI-OBJECTIVE GERABOX DESIGN

Among the most widely adopted MOO techniques are evolutionary algorithms (EAs), which excel at exploring complex search spaces without the need for derivative information. One of the most influential frameworks is the Non-dominated Sorting Genetic Algorithm II (NSGA-II), introduced by Deb et al. [1]. This algorithm utilizes fast non-dominated sorting, elitism, and crowding distance mechanisms to generate a well-distributed set of Pareto-optimal solutions. NSGA-II has since become a benchmark tool for solving engineering design problems, as detailed in foundational works on evolutionary multi-objective optimization [2, 3].

Numerous studies have applied NSGA-II to gearbox design problems. For instance, Deb and Jain [4] investigated multi-speed gearbox design using NSGA-II to balance multiple performance objectives. Patil et al. [5] applied NSGA-II to a two-stage spur gearbox, optimizing for both efficiency and weight. Le Duc Bao et al. [6] introduced the MARCOS for Multi-Objective Optimization of a Two-stage Helical Gearbox with Double Gears in the First Stage. Ngoc Pi Vu et al. [7] presented the new approach, using Optimizing Two-Stage Gear Design using NSGA-II with MATLAB: Multi-Objective Approach on Mass and Efficiency Trade-Off.

In the aerospace sector, Kim et al. [8] used NSGA-III (an extension of NSGA-II) to optimize angular contact ball bearings in high-speed gearboxes. In the context of planetary gear systems, Nasr et al. [9] implemented a genetic algorithm-based MOO approach to optimize gear train parameters under various load conditions. These studies demonstrate the robustness and versatility of evolutionary algorithms in solving complex, multi-objective gearbox design problems.

1.2. NON – EVOLUTIONARY AND HYBRID OPTIMIZATION APPROACHES

In contrast to purely evolutionary strategies, other researchers have employed non-evolutionary or hybrid approaches for gearbox optimization. Wei et al. [10] utilized topology optimization to design large marine gearboxes, focusing on structural layout efficiency. Qi et al. [11] combined response surface methodology (RSM) with acoustic participation analysis to optimize gearbox noise and vibration. Similarly, Xiang et al. [12] introduced a layered optimization strategy for magnetic planetary gearboxes in hybrid powertrains, demonstrating

a hierarchical design process. Zhao et al. [13] employed deep learning-based multi-objective diagnosis, showcasing the integration of intelligent data-driven methods into gearbox health assessment and optimization.

These alternative techniques offer valuable insights and highlight the evolving landscape of gearbox optimization beyond conventional evolutionary methods.

1.3. DECISION-MAKING-ORIENTED AND MCDM-BASED GERABOX OPTIMIZATION

Another line of research has focused on applying multi-criteria decision-making (MCDM) methods in conjunction with optimization procedures. Dinh et al. explored the design of two-stage helical gearboxes using both the MARCOS method [14] and integrated MCDM frameworks [15], aiming to rank and select optimal solutions based on predefined criteria. Similarly, Thanh et al. [16] addressed the trade-off between efficiency and bottom area by combining engineering modeling with MCDM for decision support.

While MCDM-based strategies are useful for ranking alternatives, they typically require external weighting schemes and may not fully capture the continuous trade-off surface between objectives. This limitation highlights the need for a Pareto-based optimization approach that does not rely on predefined weights or post-processing.

In recent years, the integration of digital engineering tools and decision-support methodologies has further expanded the scope of gearbox design research. Rjabtsikov et al. [17] introduced a parametric digital twin framework for autonomous electric vehicle transmissions, highlighting the importance of model-based design and performance prediction in modern powertrain systems. In the context of industrial robotics, Yaqoob et al. [18] proposed a gear selection tool aimed at improving accuracy and transmission matching, emphasizing the need for systematic and data-driven gearbox configuration strategies. Furthermore, Dinh et al. [19] applied the TOPSIS method to solve the multi-objective optimization problem of a two-stage helical gearbox with double gear-sets in the first stage, demonstrating the applicability of multi-criteria decision-making techniques in gearbox parameter selection. These studies collectively underline the growing demand for optimization-based, predictive, and decision-support-oriented approaches in transmission system design, particularly for compact and efficiency-sensitive applications.

1.4. MOTIVATION AND CONTRIBUTION OF THIS STUDY

Although substantial progress has been made, most existing studies either:

1. focus on single-stage or spur gear systems,
2. require post-optimization decision-making (e.g., via MCDM), or
3. lack detailed Pareto trade-off analysis between compactness and efficiency in two-stage helical gearboxes.

To bridge these gaps, this study proposes a direct multi-objective optimization approach using NSGA-II, targeting the simultaneous:

- Minimization of total cross-sectional area, representing structural compactness,
- Maximization of transmission efficiency, critical for reducing energy loss and improving operational performance.

By leveraging the proven strengths of NSGA-II and building upon the insights from prior works across evolutionary, hybrid, and decision-making-based methodologies, this research delivers a comprehensive Pareto frontier that empowers engineers with flexible design choices in the early stages of gearbox development.

2. OPTIMIZATION PROBLEM FORMULATION

2.1. DETERMINING GERABOX BOTTOM AREA

For a two stage helical gearbox, the cross-sectional area A_c is determined by equation (1) (see Fig. 1).

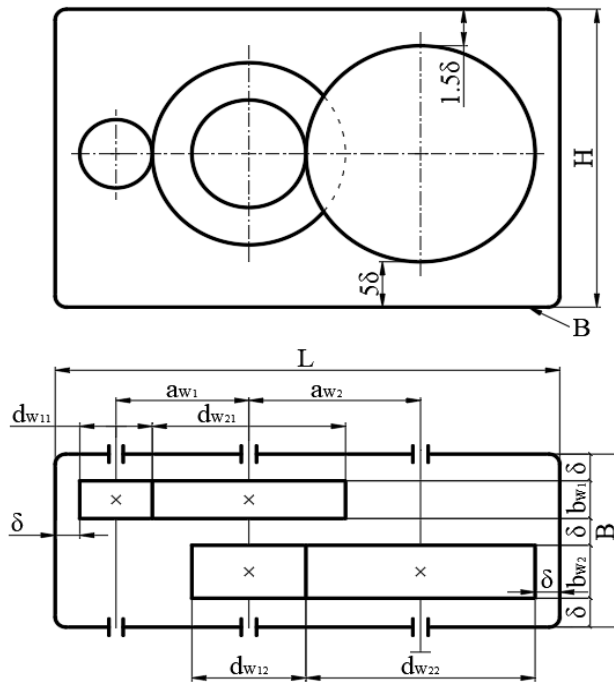


Fig. 1. Schema for calculation of gearbox cross-sectional area

$$A_c = (L \cdot H) \tag{1}$$

Where L and H are calculated by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \tag{2}$$

$$H = \max(d_{w21}, d_{w22}) + 6.5 \cdot \delta \tag{3}$$

Where, $\delta = 7 \div 10$ (mm) [20]; d_{w1i} , d_{w2i} ($i = 1 \div 2$) are the pitch diameter of the pinion and the gear of stage i which can be computed by [21]:

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (4)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (5)$$

In the above Equations, a_{wi} ($i = 1 \div 2$) is the center distance of stage i ; a_{wi} can be determined by [21]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (6)$$

In (7), X_{bai} is the wheel face width coefficient of stage i th; T_{1i} ($i = 1 \div 2$) is the pinion torque of stage i and it can be computed by:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^3 (u_j \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \quad (7)$$

2.2. DETERMINING GERABOX EFFICIENCY

The gearbox efficiency (%) can be determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (8)$$

In which, P_l is the total gearbox power loss which is found by [21]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{z0} \quad (9)$$

In (10), P_{lg} , P_{lb} , P_{ls} , and P_{z0} are the power loss in the gears, in bearings, in seals, and in the idle motion. These components are computed as in [17].

2.3. OBJECTIVES FUNCTIONS

This work formulates the optimization problem as a bi-objective minimization issue, representing two critical performance metrics of a two-stage helical gearbox:

– Minimizing the gearbox cross-sectional area:

$$\min f_1(X) = A_c \quad (10)$$

– Maximizing the gearbox efficiency:

$$\min f_2(X) = \eta_{gb} \quad (11)$$

The design variable vector X includes the key geometrical and performance-related parameters of the gearbox. Five parameters are typically employed to define the geometry: u_1 , X_{ba1} , X_{ba2} , AS_1 , and AS_2 [22]. Based on research findings [22], the optimal values of AS_1

and approach their maximum allowable values. As a result, the three most sensitive and adjustable parameters are identified as decision variables: u_1, X_{ba1} . The following is established:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (12)$$

2.4. CONSTRAINS

For a two stage helical gearbox, $u_i=1 \div 9$; $X_{bai} = 0.25 \div 0.4$ ($i=1 \div 2$) [21]. The MOOP includes the subsequent limitations:

$$1 \leq u_i \leq 9 \quad (13)$$

$$0.25 \leq X_{bai} \leq 0.4 \quad (14)$$

3. OPTIMIZATION METHODOLOGY

3.1. OPTIMIZATION APPROACH USING NSGA-II

To solve this problem, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed due to its proven performance in handling conflicting objectives and generating well-distributed Pareto fronts [1]. The main features of NSGA-II include:

- Fast non-dominated sorting for population ranking.
- Crowding distance calculation for diversity preservation.
- Elitism to retain the best solutions over generations.

For each fixed u_h , the algorithm searches the design space for optimal combinations of u_1, X_{ba1}, X_{ba2} . The implementation has been developed using MATLAB, incorporating the parameters outlined in Table 1.

Table 1. NSGA-II Algorithm Parameters Used in MATLAB Implementation

Parameter	Value
Population size	100
Generations	200
Crossover rate	0.9
Mutation rate	0.1
Selection method	Binary tournament
Encoding	Real-valued

3.2. MULTI-SCENARIO OPTIMIZATION BY SWEEPING u_h

To investigate the influence of the total transmission ratio u_h on gearbox performance, a multi-scenario optimization strategy is employed. Specifically, the NSGA-II algorithm is executed independently for a set of pre-defined transmission ratios:

$$u_h \in \{5, 10, 15, 20, 25, 30, 35\} \tag{15}$$

For each scenario, the optimization is carried out with the same algorithmic settings, objective functions, and constraints, while treating u_h as a fixed parameter.

This strategy allows for a systematic exploration of how increasing transmission demands affect the shape and range of the Pareto fronts between efficiency and structural compactness. Unlike single-scenario studies, this approach provides a more complete perspective on the design trade-offs across application contexts.

The resulting Pareto fronts for all u_h values are later visualized and compared in Fig. 1, and their implications are analysed in detail in Section 4. This separation ensures that the methodology remains general and scalable, while the discussion of trends and engineering interpretation is deferred to the results section.

By aggregating optimal solutions from multiple scenarios, this framework also supports the development of empirical design trends—such as the linear relationship between the stage-1 gear ratio u_1 and total ratio u_h —which can accelerate future design workflows and reduce the need for repeated optimization.

4. RESULTS AND DISCUSSIONS

4.1. TRADE-OFF BEHAVIOUR UNDER VARYING TRANSMISSION RATIOS

Figure 2 illustrates the Pareto fronts obtained across different total transmission ratios (u_h), revealing a consistent pattern: as efficiency improves, the gearbox cross-sectional area increases. Notably, the shape and height of the Pareto fronts degrade with increasing u_h , indicating that higher transmission ratios limit the ability to achieve both compact and efficient designs.

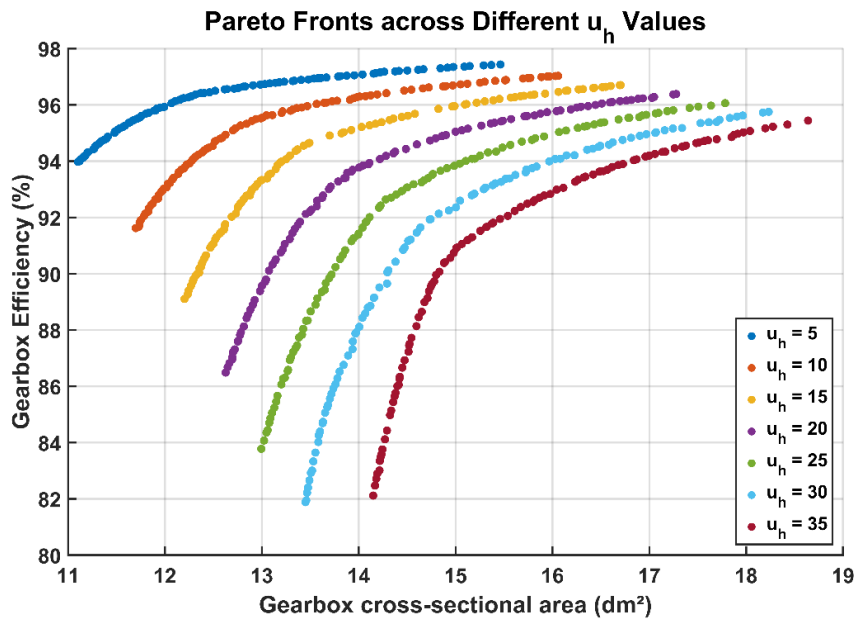


Fig .2. Pareto Fronts across Varying u_h Values

This trend contrasts with results by Patil et al. [5] on spur gearboxes, where the deterioration in efficiency was less significant due to lower gear interaction losses. Compared to the MCDM-based optimization in Dinh et al. [14], which yielded single optimal points per scenario, this study provides a full Pareto front, granting engineers greater flexibility in selecting optimal trade-offs.

Moreover, while Qi et al. [11] explored trade-offs between vibration and performance using RSM, they did not provide a comprehensive view of the structural impact. In contrast, this work explicitly quantifies how efficiency loss is associated with structural growth, providing a clearer map for design decision-making.

4.2. OPTIMAL CONFIGURATIONS AND EFFICIENCY-COMPACTNESS RELATIONSHIP

Optimal design configurations were extracted using two strategies: (i) minimum cross-sectional area, and (ii) maximum efficiency-to-area ratio. These configurations consistently demonstrate that increasing u_h leads to larger gearbox dimensions and a decline in efficiency—mirroring the trend shown in Fig. 3.

Trend of Mean Cross-sectional area and Efficiency vs. Transmission Ratio (u_h)

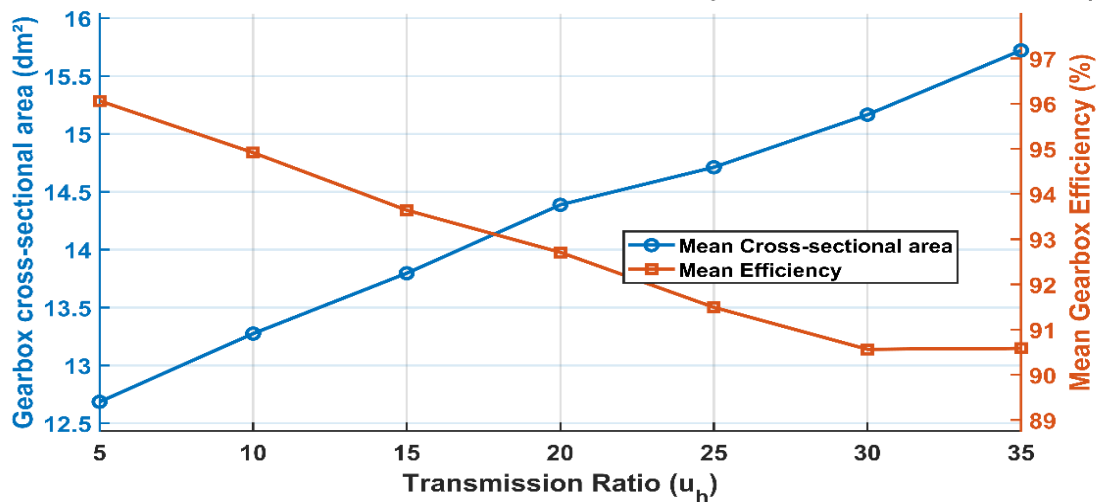


Fig. 3. Trend of Mean Bottom Area and Efficiency vs. Gearbox Ratio (u_h)

The efficiency drop—from 97.4% to 90.3% as u_h increases from 5 to 35—highlights an important engineering constraint. This decline is more gradual and better managed in our NSGA-II approach than in the MCDM-based work by Thanh et al. [16], which showed steeper efficiency degradation without an explicit trade-off structure. Furthermore, compared with Nasr et al. [9], who optimized planetary gear trains using GA and reported limited flexibility in high u_h , this study demonstrates broader adaptability via ratio decomposition and torque splitting.

These findings are significant for applications such as robotics or electric drivetrains, where space is limited and efficiency is critical. By providing a continuous solution space, this method supports more nuanced engineering compromises.

4.3. EFFICIENCY LOSS VERSUS STRUCTURAL GROWTH: A STRATEGIC BALANCE

One of the most practically valuable outcomes is the ability to visualize and quantify the balance between efficiency and size. The mean trend observed in Fig. 2 shows a clear inverse relationship between the two objectives. Unlike Wei et al. [10], who applied topology optimization on marine gearboxes primarily focusing on structural layout, this study integrates both structure and performance in a unified optimization framework.

The resulting Pareto fronts demonstrate that moderate transmission ratios (e.g., $u_h=10$ to 20) offer a desirable compromise: compact designs (13–14.5 dm²) while maintaining high efficiencies (~92–94%). This provides a design guideline for early-stage gearbox sizing, especially in multidisciplinary design environments.

4.4. DESIGN VARIABLE BEHAVIOUR AND RATIO DECOMPOSITION STRATEGY

Figure 4 presents a strong linear relationship between stage-1 ratio u_1 and total transmission ratio u_h (with $R^2=0.9773$), captured by the following regression:

$$u_1 = 0.1798 \cdot u_h + 1.6953 \tag{16}$$

This finding allows rapid estimation of u_1 without re-optimization for each u_h , which is a useful feature not discussed in previous works such as Deb and Jain [4] or Gologlu and Zeyveli [23], who treated ratios as static inputs.

The design variable X_{iba1} , representing torque distribution in the first stage, tends to shift toward lower values (0.25–0.30) for high-efficiency designs (Table 2). This suggests an intelligent torque-splitting strategy that reduces losses in early transmission stages. Compared to the MARCOS-based results of Dinh et al. [14], which did not explore internal power flow in depth, this study brings to light a critical design insight for achieving efficiency without structural overload.

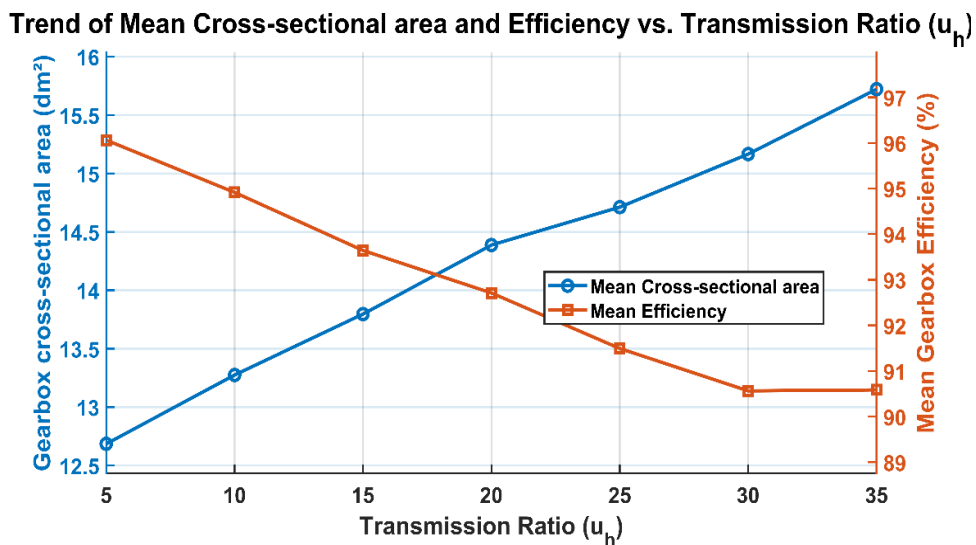


Fig. 4. Linear Regression of u_1 vs. Gearbox Ratio (u_h)

Table 2: Optimal Gearbox Designs at Different Transmission Ratios u_h

u_h	u_1	u_2	X_{iba1}	X_{iba2}	Ac (dm ²)	η (%)
5	1.27	3.95	0.25	0.40	15.46	97.43
10	2.04	4.91	0.25	0.40	16.06	97.03
15	2.66	5.63	0.25	0.40	16.70	96.70
20	3.22	6.21	0.25	0.40	17.28	96.38
25	3.73	6.70	0.25	0.40	17.78	96.06
30	4.21	7.13	0.25	0.40	18.23	95.75
35	4.66	7.51	0.25	0.40	18.64	95.44

Significance of this study:

By integrating NSGA-II with a detailed mechanical model and focusing on dual-objective optimization (efficiency vs. size), this study contributes a complete, data-driven framework for helical gearbox design. It extends beyond previous works by:

- Providing a continuous Pareto front instead of single-point solutions,
- Offering predictive models for gear ratio decomposition,
- Uncovering the role of torque allocation variables like X_{iba1} in managing power loss.

These insights are not only theoretically valuable, but also offer practical utility for mechanical engineers dealing with constrained spaces, energy efficiency demands, and high transmission requirements.

4.5. IMPLICATIONS FOR ENGINEERING PRACTICE

The results obtained in this study provide practical guidance for both educational and industrial applications.

In mechanical engineering education, the derived Pareto fronts and the linear relationship between the first-stage ratio u_1 and the total transmission ratio u_h can be directly applied in Machine Elements course projects. Students can use these results to determine reasonable partial gear ratios and evaluate the trade-off between gearbox compactness and efficiency without performing complex optimization procedures.

From an industrial perspective, the proposed framework supports preliminary design calculations for two-stage helical gearboxes in small-batch or single-piece production, where rapid decision-making is essential. The predictive ratio-decomposition model enables engineers to estimate near-optimal configurations quickly, reducing iterative trial-and-error calculations while maintaining acceptable efficiency and structural compactness.

Overall, the study offers a practical design reference that bridges optimization theory with real engineering practice.

5. CONCLUSION

This study presented a Pareto-based multi-objective optimization framework for two-stage helical gearbox design using NSGA-II. The main conclusions are summarized as follows:

– Methodological novelty: A direct Pareto-front-based optimization approach was implemented to simultaneously minimize gearbox cross-sectional area and maximize mechanical efficiency, without relying on post-processing decision-making methods (e.g., MCDM weighting). This provides a continuous solution space rather than a single optimal configuration. Quantified trade-off behaviour: The results demonstrate a clear inverse relationship between structural compactness and efficiency. As the total transmission ratio u_h increases from 5 to 35: The cross-sectional area increases from 15.46 dm² to 18.64 dm² ($\approx 20.6\%$ growth); The efficiency decreases from 97.43% to 95.44% ($\approx 2\%$ absolute reduction). This quantifies the structural penalty required to maintain high transmission ratios.

– Identification of an optimal operating range: Moderate transmission ratios ($u_h = 10\text{--}20$) were found to provide the best balance, achieving efficiencies of 92–97% while keeping the cross-sectional area within 13–17 dm². This range represents the most favorable engineering compromise.

– Progress in understanding gearbox ratio decomposition: A strong linear relationship between the first-stage ratio and total transmission ratio was identified ($R^2=0.9773$), enabling predictive estimation of $u1$ without repeated optimization. This enhances understanding of ratio distribution mechanisms in two-stage helical gearboxes.

– Insight into torque allocation strategy: The torque-splitting parameter X_{bal} consistently converges toward its lower bound (0.25–0.30) in high-efficiency designs, indicating that conservative torque allocation in the first stage reduces early power losses. This provides new insight into internal power-loss control strategies.

– Practical implications: The derived Pareto fronts and predictive ratio-decomposition model can be directly applied in machine elements course projects and preliminary gearbox sizing for small-batch or single-piece production, reducing iterative trial-and-error calculations.

– Limitations of the study: The analysis focuses on geometric compactness and mechanical efficiency only. Acoustic performance, fatigue life, manufacturability constraints, and cost factors were not considered. Additionally, the study employed a standard NSGA-II framework without comparative analysis against other evolutionary algorithms or convergence metrics.

– Future research directions: Future work should integrate additional performance criteria (noise, durability, thermal effects), compare alternative multi-objective algorithms, and explore surrogate modelling techniques to improve computational efficiency.

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