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MULTI-OBJECTIVE OPTIMIZATION OF TWO-STAGE HELICAL GEARBOXES WITH DUAL-GEAR FIRST STAGE USING NSGA-II AND TOPSIS: MINIMIZING VOLUME AND MAXIMIZING EFFICIENCY

This study presents a multi-objective optimization framework for the design of two-stage helical gearboxes with a dual-gear first stage. The optimization aims to simultaneously minimize the gearbox volume and maximize the transmission efficiency, two inherently conflicting objectives. The Non-dominated Sorting Genetic Algorithm II (NSGA-II) was employed to generate Pareto-optimal solutions, capturing the trade-off between compactness and efficiency. Subsequently, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method was applied to rank the Pareto solutions and select the most balanced design. The results reveal clear trends: larger transmission ratios tend to reduce efficiency while increasing gearbox volume, and the optimal compromise strongly depends on the ratio distribution between stages. Pareto fronts across different transmission ratios demonstrate the volume–efficiency trade-off, while TOPSIS effectively identifies design points that balance both objectives. The proposed hybrid approach provides a systematic methodology for designing compact and energy-efficient gearboxes, offering valuable insights for practical engineering applications.

1. INTRODUCTION

The design of gearboxes plays a central role in modern mechanical systems, where compactness and energy efficiency are critical requirements. Helical gearboxes, in particular, are widely applied in industrial transmission systems due to their smooth operation, high load-carrying capacity, and favourable efficiency. However, gearbox design is inherently a multi-

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objective problem, as reducing volume often conflicts with maximizing efficiency. This trade-off calls for advanced optimization strategies capable of balancing conflicting performance measures.

Multi-objective evolutionary algorithms (MOEAs) have been extensively developed to address such challenges. Among them, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is one of the most prominent methods, offering fast convergence and elitism to maintain solution diversity [1]. Its foundations are well-established in the literature of evolutionary computation for multi-objective problems [2, 3]. Numerous studies have demonstrated the suitability of NSGA-II for solving mechanical and gearbox design problems, showing its ability to produce Pareto-optimal fronts that capture trade-offs among design objectives [4–6].

In the context of gearbox optimization, early studies focused on deterministic or single-objective formulations. For instance, Wang and Wang optimized spur gear sets with respect to engineering design parameters [7], while Gologlu and Zeyveli introduced a genetic approach for automated gear drive design [8]. Subsequent works expanded these ideas by applying population-based optimization algorithms such as particle swarm optimization and simulated annealing for weight and dimension reduction in gear trains [9]. More recent studies incorporated tribological aspects, enhancing the realism of gearbox optimization frameworks [10].

The application of evolutionary algorithms specifically for gearbox optimization has been a focus of comparative analyses. Méndez et al. investigated the performance of different evolutionary algorithms in gear system optimization, highlighting their efficiency in finding global optima [11]. Similarly, Chong demonstrated that genetic algorithms could effectively handle the nonlinear design space of gear trains [12], while Daoudi et al. extended the approach to multi-objective optimization of epicyclical gear trains [13]. These efforts underline the versatility of evolutionary algorithms in gearbox-related problems.

For helical gearboxes, NSGA-II has been directly applied to multi-objective optimization scenarios. Sanghvi et al. optimized a two-stage helical gear train using NSGA-II, targeting simultaneous improvements in design efficiency [4]. Patil et al. explored two-stage spur gearboxes using NSGA-II with tribological considerations [5], whereas Maputi and Arora combined NSGA-II with decision-making methods for two-stage spur gearbox optimization [6]. Such hybrid approaches highlight the need for systematic frameworks that not only generate Pareto solutions but also provide decision support for selecting the most practical design.

Recent works have emphasized integrating multi-criteria decision-making (MCDM) methods with MOEAs to identify the best compromise solutions. For example, Dinh et al. applied the MARCOS method for two-stage helical gearboxes [14] and employed other MCDM methods to address conflicting objectives [15]. Hung and Huong optimized a two-stage helical gearbox with the aim of improving efficiency and reducing height [16]. Likewise, Xin et al. proposed a multi-objective design approach for planetary gear reducers in large mining applications [17]. In addition, Dinh et al. specifically applied TOPSIS to solve the optimization of two-stage helical gearboxes with first-stage double gear sets, underscoring the effectiveness of integrating evolutionary optimization with decision-making frameworks [18].

Despite these advances, important research gaps remain. Most existing studies have focused on spur gear or planetary gear systems [9, 19], with fewer addressing two-stage helical gearboxes with dual-gear first stages, which represent a unique design configuration with significant potential for balancing compactness and efficiency. Furthermore, while MOEAs such as NSGA-II generate Pareto fronts effectively, the challenge of selecting the best compromise solution for engineering implementation persists. The integration of NSGA-II with a robust decision-making method like TOPSIS offers a promising yet underexplored direction for gearbox design optimization.

In this study, we propose a hybrid NSGA-II and TOPSIS framework for the multi-objective optimization of two-stage helical gearboxes with a dual-gear first stage. The research aims to simultaneously minimize gearbox volume and maximize efficiency, two objectives that are inherently conflicting. By analyzing Pareto fronts across different transmission ratios and applying TOPSIS to identify the most balanced solutions, the study provides new insights into the trade-off between compactness and energy efficiency. This work contributes both a methodological advancement and practical design guidelines for compact, high-efficiency gearbox applications.

2. OPTIMIZATION PROBLEM

2.1. CALCULATION OF GEARBOX VOLUME

For a two stage helical gearbox with two gears in the first stage, the volume *V* can be found by (Fig. 1):

$$V = (L \cdot B \cdot H) \tag{1}$$

Where L, B and H are calculated by:

$$L = d_{w11} + \frac{d_{w21}}{2} + \frac{d_{w12}}{2} + d_{w22} + 4 \cdot \tag{2}$$

$$B = 2 \cdot b_{w1} + b_{w2} + 7 \cdot \delta \tag{3}$$

$$H = \max\left(d_{w21}, d_{w22}\right) + 8.5 \cdot \delta \tag{4}$$

In which, δ = 7÷10 (mm) [20]; d_{w1i}, d_{w2i} (i = 1÷2) represent the pitch diameter of the pinion and the gear of stage i which are computed by [20]:

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \tag{5}$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \tag{6}$$

Where, a_{wi} (i = 1÷2) is the center distance of stage I which is found by [20]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \tag{7}$$

In which, X_{bai} is the wheel face width coefficient of stage i$_{th}$; T_{1i} (i = 1÷2) is the pinion torque of stage i which is computed by:

$$T_{11} = \frac{T_r}{2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \tag{8}$$

$$T_{12} = \frac{T_r}{u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \tag{9}$$

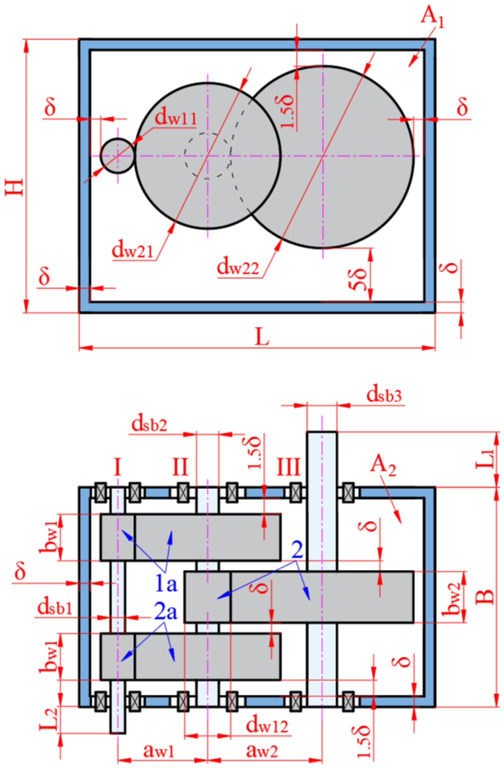

Fig. 1. Schema for determination of gearbox volume

2.2. DETERMINING GEARBOX EFFICIENCY

The total efficiency of the gearbox ηgb can be determined using the following equation:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{10}$$

In which, Pl is the total power loss in the gearbox and Pin is the input gearbox power. The total loss Pl of the gearbox can be found by [21]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \tag{11}$$

Where, P_{lg}, P_{lb}, P_{ls}, and P_{Z0} denote gear meshing loss, bearing friction loss, seal resistance loss, and loss due to idle motion, respectively. These components are determined as reported in [22].

2.3. OBJECTIVE FUNCTIONS AND CONSTRAINTS

The design of the two-stage helical gearbox with a split first stage is formulated as a bi-objective optimization problem, where two conflicting objectives are addressed simultaneously: minimizing the gearbox volume V and maximizing the mechanical efficiency ηgb. Accordingly, the objective functions are expressed as:

$$min f_1 = V \tag{12}$$

$$max f_2 = \eta_{gb} \tag{13}$$

The optimization problem is formulated with three continuous design variables that critically affect both the geometric configuration and performance of the gearbox. Specifically, $u1$, X_{ba1}, and X_{ba2}. These variables determine the allocation of gear ratios and the dimensional characteristics of each stage, thereby exerting a direct influence on the gearbox volume and transmission efficiency. The following practical limitations apply to the design variables in order to guarantee feasibility and manufacturability [20]:

The optimization framework considers three continuous design variables that have a critical impact on both the geometric configuration and the performance of the gearbox, namely the gear ratio of the first stage (u_1), and the addendum modification coefficients of the first and second stages (X_{ba1}, X_{ba2}). These parameters directly influence the distribution of transmission ratios and the dimensional characteristics of each stage, thereby determining the overall gearbox volume and efficiency. To ensure feasibility and manufacturability, the design variables are bounded within the following practical limits [20]:

$$1 \le u_{\mathrm{i}} \le 9 \tag{14}$$

$$0.25 \le Xba_{\mathrm{i}} \le 0.4 \tag{15}$$

3. OPTIMIZATION METHODOLOGY

3.1. NSGA II METHOD

The NSGA-II is one of the most widely applied evolutionary algorithms for solving multi-objective optimization problems due to its efficiency in generating diverse Pareto-optimal solutions [1]. The core idea of NSGA-II lies in simultaneously optimizing multiple conflicting objectives by maintaining a set of non-dominated solutions instead of a single global optimum.

In NSGA-II, the population evolves through the operators of selection, crossover, and mutation, similar to traditional genetic algorithms. However, the algorithm introduces several improvements that enhance its performance for multi-objective optimization:

- Fast non-dominated sorting: The entire population is sorted into different Pareto fronts based on dominance relationships. Solutions in the first front are non-dominated by any other solutions, while those in subsequent fronts are dominated by at least one solution.

- Crowding distance calculation**:** To preserve solution diversity across the Pareto front, NSGA-II computes a crowding distance measure for each solution, ensuring uniform distribution and avoiding premature convergence.
- Elitism: The combined population of parents and offspring is sorted, and the best solutions are preserved for the next generation, ensuring that high-quality solutions are not lost.

Through these mechanisms, NSGA-II produces well-distributed Pareto fronts, which represent the trade-off between conflicting objectives—in this case, minimizing gearbox volume and maximizing efficiency. The algorithm is particularly suitable for gearbox design problems due to its ability to navigate complex and nonlinear design spaces.

3.2. TOPSIS METHOD

The TOPSIS method was used to solve the multi-criteria decision-making (MCDM) problem in this study. To use the TOPSIS technique efficiently, it is important to perform the following processes in a systematic order [23]:

+) Construct the decision matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \tag{16}$$

where x_{mn} is the value of criterion *n* in variant *m*.

+) Normalize the decision matrix:

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{17}$$

+) Compute the weighted normalized decision matrix

$$l_{ij} = w_j \times k_{ij} \tag{18}$$

In which, w_j is the weight of the j^{th} criterion.

+) Identify the best and worst alternatives

$$A^+ = \{l_1^+, l_2^+, \ldots, l_j^+, \ldots, l_n^+\} \tag{19}$$

$$A^- = \{l_1^-, l_2^-, \ldots, l_j^-, \ldots, l_n^-\} \tag{20}$$

where l_j^+ and l_j^- denote the best and worst values of the j criterion, respectively (j = 1,2, .., n).

+) Calculate the Euclidean distances to the ideal and negative-ideal solutions:

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(l_{ij} - l_j^+\right)^2} \tag{21}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}\left(l_{ij} - l_j^-\right)^2} \tag{22}$$

where $i = 1, 2, \ldots, m$.
+) Determine the relative closeness of each option to the ideal solution:

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{23}$$

where i = 1, 2, …, m; $0 \le R_i \le 1$.
+) Rank the alternative’s order by maximizing *R*.

4. RESULTS AND DISCUSSION

The optimization of the two-stage helical gearbox with a dual-gear first stage was carried out using NSGA-II to generate Pareto-optimal solutions, followed by the application of TOPSIS for selecting the most balanced design. The results are presented in three stages: (*i*) regression analysis of the relationship between the ratio allocation $u1$ and the equivalent transmission ratio u_h, (*ii*) trend analysis of gearbox performance metrics as functions of u_h, and (*iii*) multi-objective trade-off evaluation through Pareto fronts and TOPSIS-based decision making.

4.1. OPTIMAL SOLUTIONS AND REGRESSION ANALYSIS

The Pareto-optimal sets generated by NSGA-II were further processed using the TOPSIS method to identify compromise solutions between gearbox volume minimization and efficiency maximization. Table 1 presents the TOPSIS-based optimal results for different u_h, including *u1*, *Xba1* and *Xba2* , gearbox volume (*V*), and efficiency (*ηgb*).

From these results, it can be observed that the gearbox volume steadily increases as u_h grows, while the efficiency gradually decreases. For instance, when u_h = 5, the gearbox achieves the smallest volume ($2.09 \times 10^7 mm^3$) with the highest efficiency (94.82%). In contrast, for u_h = 35, the gearbox becomes more compact in terms of ratio distribution but suffers from lower efficiency (89.73%). This reflects the intrinsic trade-off between compactness and performance.

To further analyse the allocation of the first-stage gear ratio, the optimal values of $u1$ corresponding to each u_h were fitted using a linear regression model. The regression equation is:

$$u_1 = 0.1970 \cdot u_h + 2.3871 \tag{24}$$

with a determination coefficient of R^2=0.9788.

Table 1. Optimal results from TOPSIS

u_h	u_1	X_{ba1}	X_{ba2}	V (mm^3)	Efficiency (%)
5	3.07	0.25	0.40	20898024.20	94.82
10	4.24	0.25	0.40	22534468.22	94.18
15	5.59	0.25	0.40	23624744.31	93.06
20	6.59	0.25	0.40	24727983.30	92.12
25	7.53	0.25	0.40	25708649.13	91.03
30	8.49	0.25	0.40	26521551.48	89.98
35	8.77	0.25	0.40	27836660.68	89.73

This high R^2 confirms that the linear model effectively captures the dependency of u_1 on u_h. Figure 2 illustrates the regression fit alongside the optimal data points, showing close agreement. The regression equation provides a simple yet accurate expression for determining the first-stage ratio based on the overall transmission ratio.

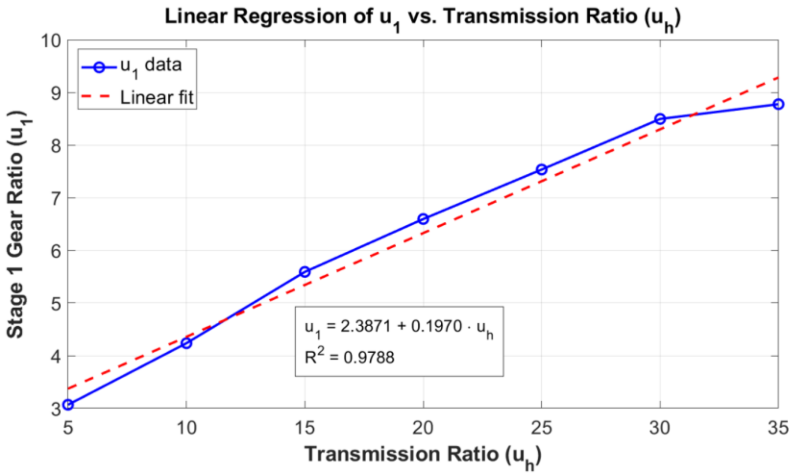

Fig. 2. Linear regression of u_1 versus u_h from TOPSIS-based optimal solutions

It is noteworthy that the optimal values of $X_{ba1} = 0.25$ and $X_{ba2} = 0.40$ remain consistent across all u_h. This suggests that these geometric coefficients are relatively insensitive to changes in the transmission ratio once optimal solutions are identified, and they can be considered robust parameters in gearbox design.

4.2. TREND OF GEARBOX VOLUME AND EFFICIENCY

The mean trends of gearbox volume and efficiency as functions of u_h are illustrated in Fig. 2. A clear and opposing tendency is observed:

- Gearbox volume (V) increases monotonically with larger u_h. This is primarily due to the enlargement of gear dimensions and shaft spacing required to achieve higher transmission ratios, which leads to an increase in the overall gearbox housing volume.

- Gearbox efficiency (η_{gb}), on the other hand, decreases as u_h grows. Higher transmission ratios typically cause greater power losses arising from additional meshing, sliding friction, and churning effects in the lubricant, which collectively reduce the efficiency.

For example, when u_h rises from 5 to 35, the gearbox volume increases by more than 30%, whereas efficiency drops by approximately 5 percentage points. This divergent behavior highlights the inherent conflict between compactness and performance, thereby reinforcing the need for multi-objective optimization.

Figure 3 demonstrates that although very small values of u_h provide high efficiency, they fail to achieve significant volume reduction. Conversely, large values of u_h can compact the design but at the expense of efficiency loss. Thus, selecting an appropriate compromise solution requires balancing both trends, which justifies the integration of NSGA-II with the TOPSIS decision-making approach.

Fig. 3. Mean trends of gearbox volume and efficiency as functions of u_h

4.3. PARETO- OPTIMAL FRONTS ACROSS DIFFERENT U_H

The Pareto fronts generated by NSGA-II for different values of u_h are presented in Figure 3. Each curve represents the trade-off boundary between gearbox volume and efficiency under a specific ratio allocation.

The results highlight several key insights:

- Non-dominated solutions: For every u_h, the Pareto front clearly illustrates that no single solution can simultaneously minimize gearbox volume and maximize efficiency. Instead, a set of non-dominated alternatives exist, where improving one objective necessarily worsens the other.
- Impact of *u*h: At smaller transmission ratios (u_h = 5–10), efficiency is relatively high, but the achievable reduction in volume is limited. Conversely, for larger transmission ratios (u_h = 25–35), the gearbox volume is significantly reduced, but efficiency declines sharply.
- Shape of the trade-off: The Pareto fronts exhibit a convex form, typical in engineering optimization problems, which indicates diminishing returns: once efficiency is maximized, further improvements in compactness result in disproportionately large losses in efficiency.

For instance, at $u_h = 20$, a balanced trade-off can be achieved, where the gearbox volume is reduced compared with lower u_h cases, while efficiency remains above 92%. At u_h=30, although the gearbox becomes more compact, efficiency falls below 90%, which may be unacceptable for practical applications where energy savings are critical.

These findings emphasize that Pareto fronts serve as an effective visualization of design trade-offs, providing designers with a spectrum of feasible options. However, the final selection of an optimal point requires a systematic decision-making process, such as TOPSIS, to ensure that the chosen solution represents the most balanced compromise between conflicting objectives.

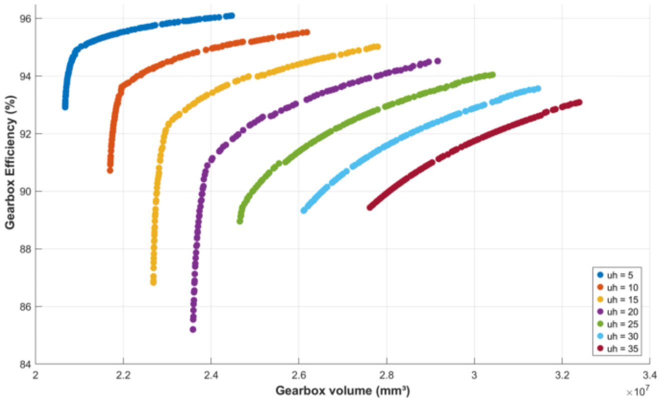

Fig. 4. Pareto fronts of gearbox volume and efficiency for different values of *uh*

5. CONCLUSIONS

This study proposed a hybrid multi-objective optimization framework for the design of two-stage helical gearboxes with a dual-gear first stage. The main findings and contributions of this work can be summarized as follows:

- A novel hybrid optimization framework combining NSGA-II and the TOPSIS decision-making method was developed for the design of two-stage helical gearboxes with a dual-gear first stage. The proposed approach enables the simultaneous optimization of two conflicting objectives: minimizing gearbox volume and maximizing transmission efficiency.
- The optimization results revealed a clear quantitative trade-off between gearbox compactness and efficiency. When the overall transmission ratio u_hincreased from 5 to 35, the gearbox volume increased from approximately 2.09×10^7 mm^3 to 2.78×10^7 mm^3, while the efficiency decreased from 94.82% to 89.73%. This confirms that higher transmission ratios lead to larger gear dimensions and increased power losses.
- The analysis of the TOPSIS-based optimal solutions showed that the optimal first-stage gear ratio $u1$ strongly depends on the overall transmission ratio uh. This relationship can be accurately described by the regression equation $u1 = 0.197u_h + 2.387$ with a determination coefficient of $R^2 = 0.9788$, which provides a useful guideline for preliminary gearbox design.

- The geometric coefficients $X_{ba1} = 0.25$ and $X_{ba2} = 0.40$ remained constant across all optimized cases, indicating that these parameters are relatively insensitive to variations in transmission ratio and can be considered robust design values for this gearbox configuration.
- The generated Pareto fronts demonstrated that no single design solution can simultaneously minimize gearbox volume and maximize efficiency. Instead, designers must select a compromise solution depending on practical requirements. The integration of TOPSIS provides a systematic and effective decision-support tool for identifying such balanced design solutions.
- From a scientific perspective, this study improves the understanding of how transmission ratio allocation influences the trade-off between gearbox compactness and efficiency in two-stage helical gearboxes with a dual-gear first stage. The results provide useful design insights for the development of compact and energy-efficient mechanical transmission systems.

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