

Received: 02 June 2026 / Accepted: 14 July 2026 / Published online: 05 October 2026

*gearbox mass,
efficiency,
volume,
transmission ratio*

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MULTI-OBJECTIVE OPTIMIZATION OF TWO-STAGE HELICAL GEARBOXES USING NON-DOMINATED SORTING GENERIC ALGORITHM II (NSGA-II): MINIMIZING MASS AND VOLUME WHILE MAXIMIZING EFFICIENCY

This study presents an NSGA-II-based multi-objective optimization framework for two-stage helical gearboxes. The novelty of this work is the simultaneous optimization in case of three conflicting objectives: minimizing gearbox mass, minimizing gearbox volume, and maximizing transmission efficiency. The study does not merely optimize for a fixed set of values but examines the variation of the Pareto solution set across a very wide range of gear ratios, from 5 to 35 taking in a account the gearbox volume was calculated as $V = L \times H \times B_1$, and the total transmission ratio u_h . The optimization was implemented in MATLAB, and Pareto fronts were generated for different transmission ratios. Hypervolume, spacing, spread, and the number of Pareto solutions were used to ensure robust algorithm convergence and solution diversity. The results show unique mechanical trade-offs. While raising the total transmission ratio reduces overall gearbox bulk, it also increases necessary volume and decreases transmission efficiency. The suggested method efficiently discovers balanced Pareto-optimal solutions and provides a practical tool for designing compact and high-efficiency two-stage.

1. INTRODUCTION

Gear transmission systems are widely used in industrial machinery, automotive drivetrains, aerospace equipment, and energy systems because of their high efficiency and reliability [1, 2]. Among them, two-stage helical gearboxes are attractive due to their compact structure, smooth operation, and high load-carrying capacity. Modern gearbox design requires balancing multiple conflicting objectives, including mass reduction, compactness, and efficiency improvement [3, 4]. Traditional design methods are mainly based on deterministic calculations and single-objective optimization [1, 5], which are often inadequate for handling complex trade-offs. Consequently, multi-objective optimization techniques have gained increasing attention [6, 7]. Evolutionary algorithms, particularly NSGA-II proposed by Deb

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<https://doi.org/10.36897/jme/225926>

et al. [8], are well suited for such problems because they can efficiently generate Pareto-optimal solutions while maintaining diversity [8, 9]. NSGA-II has been successfully applied to gearbox optimization problems involving spur gears [10], planetary gear trains [11], wind turbine gearboxes [12], aeroengine gearboxes [13], and studies on efficiency, vibration, noise, and compactness improvement [14]. For two-stage helical gearboxes, Sanghvi et al. [15], Le and Vu [16], and Dinh et al. [17–20] employed NSGA-II and decision-making methods such as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Measurement Alternatives and Ranking according to Compromise Solution (MARCOS), and Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA). Other studies focused on reducing gearbox dimensions while improving efficiency [21–24]. Despite these advances, most existing studies considered only two objectives and focused mainly on gearbox dimensions. Simultaneous optimization of gearbox mass, volume, and efficiency using NSGA-II remains limited. Furthermore, the influence of the total transmission ratio on Pareto-optimal characteristics has not been comprehensively investigated. Therefore, this study proposes an NSGA-II-based multi-objective optimization framework for two-stage helical gearboxes considering three conflicting objectives: minimizing gearbox mass, minimizing gearbox volume, and maximizing transmission efficiency. The total transmission ratio is varied from 5 to 35, while hypervolume, spacing, and spread metrics are employed to evaluate optimization performance. The results provide useful insights for the design of compact and high-efficiency gearbox systems.

2. GEARBOX DESIGN FORMULATION

2.1. GEARBOX MASS DETERMINATION

The total mass of the two-stage helical gearbox is determined by:

$$m_{gb} = m_{gh} + m_g + m_s \quad (1)$$

where m_{gh} , m_g , and m_s are the housing mass, gear mass, and shaft mass, respectively. The gearbox housing mass can be calculated by:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \quad (2)$$

In which, ρ_{gh} is the housing material density and V_{gh} is the housing volume. All main dimensions can be referred to Fig. 1.

The housing volume consists of the side faces, end faces, and bottom faces:

$$V_{gh} = 2 \cdot V_{sf} + 2 \cdot V_{ef} + 2 \cdot V_{bf} \quad (3)$$

where V_{sf} is the side-face volume, V_{ef} is the end-face volume, and V_{bf} is the base-face volumes, respectively.

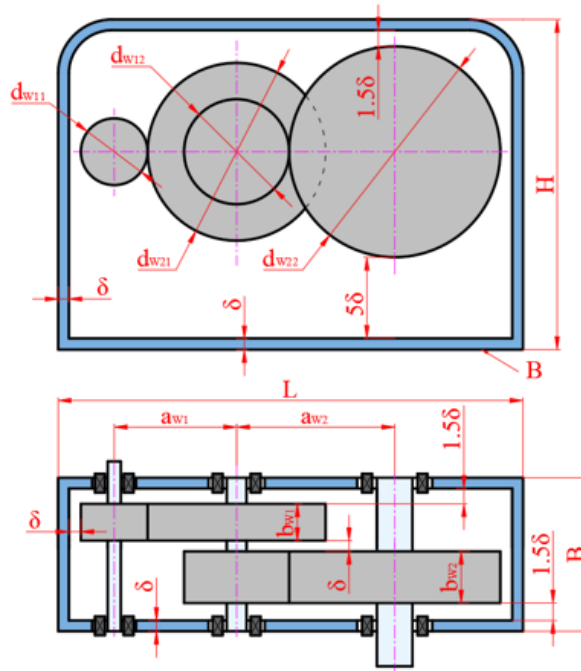


Fig. 1. Scheme for calculation of gearbox mass

The corresponding component volumes can be computed by:

$$V_{sf} = L \cdot H \cdot \delta \quad (4)$$

$$V_{ef} = B \cdot H \cdot \delta \quad (5)$$

$$V_{bf} = B \cdot L \cdot \delta \quad (6)$$

where L , B , and H are the gearbox length, width, and height, respectively, and δ is the thickness of the housing wall. The gearbox dimensions are determined by [25]:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta \quad (7)$$

$$B = b_{w1} + b_{w2} + 5 \cdot \delta \quad (8)$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot \delta \quad (9)$$

In which, d_{w11} and d_{w21} are the pitch diameters of the first-stage pinion and gear the first stage, respectively, whereas d_{w12} and d_{w22} represent those of the second-stage gear pair.

The pitch diameters can be found by:

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (10)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (11)$$

where a_{wi} is the center distance and u_i is the gear ratio of stage i .

The face widths of the two gear stages can be determined by:

$$b_{w1} = X_{ba1} \cdot a_{w1} \quad (12)$$

$$b_{w2} = X_{b2} a_{w2} \quad (13)$$

where X_{ba1} and X_{b2} are the face-width coefficients.

The centre distances are determined based on the contact strength criterion according to [25]

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([\sigma_{Hi}]^2 \cdot u_i \cdot X_{bai})} \quad (14)$$

where T_{1i} is the pinion torque and $[\sigma_{Hi}]$ is the allowable contact stress.

The torques of the two stages can be computed by:

$$T_{11} = \frac{T_r}{u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \quad (15)$$

$$T_{12} = \frac{T_r}{u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \quad (11)$$

where T_r denotes the output torque, u_{gb} denotes the total transmission ratio, and η_{hg} and η_{be} are the of the gear pair and bearing system, respectively. The total gear mass can be determined by:

$$m_g = m_{g1} + m_{g2} \quad (17)$$

The masses of the first- and second-stage gear sets are calculated in accordance with [25]:

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4} \right) \quad (18)$$

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4} \right) \quad (19)$$

where $\rho_g = 7800 \text{ (kg/m}^3\text{)}$, $e_1 = 1$ and $e_2 = 0.6$ [23].

The total shaft mass is given by:

$$m_s = \sum_{j=1}^3 m_{sj} \quad (20)$$

The mass of each shaft is calculated by:

$$m_{sj} = \rho_s \cdot \pi \cdot d_{sj}^2 \cdot l_{sj} / 4 \quad (21)$$

where $\rho_s = 7800 \text{ (kg/m}^3\text{)}$, d_{sj} denotes the shaft diameter, and l_{sj} represents the shaft length.

The shaft lengths can be found by [25]:

$$l_{s1} = B + 1.2 \cdot d_{s1} \quad (22)$$

$$l_{s2} = B \quad (23)$$

$$l_{s3} = B + 1.2 \cdot d_{s3} \quad (24)$$

The shaft diameters are determined based on the torsional strength criterion:

$$d_{sj} = [T_{1j} / (0.2 \cdot [\tau])]^{1/3} \quad (25)$$

With $[\tau] = 17 \text{ MPa}$ is the allowable shear stress [25].

2.2. GEARBOX VOLUME DETERMINATION

The housing volume of a two-stage helical gearbox is determined by (Fig. 1):

$$V_{gb} = L \cdot B \cdot H \quad (26)$$

where L , B , and H are computed by (7, 8 and 9).

2.3. GEARBOX EFFICIENCY DETERMINATION

The total gearbox efficiency η_{gb} is expressed as:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (27)$$

Where P_{in} denotes the input power and P_l represents the total power loss of the gearbox.

The total power loss is calculated as:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{z0} \quad (28)$$

where P_{lg} is the gear meshing loss, P_{lb} is the bearing friction loss, P_{ls} represents the seal resistance loss, and P_{z0} denotes the oil churning loss. These elements can be determined as reported in [16].

2.4. OBJECTIVES FUNCTIONS AND DESIGN CONSTRAINTS

The optimization of the two-stage helical gearbox is formulated as a multi-objective design problem in which three conflicting goals are considered simultaneously, including minimization of the gearbox mass m_{gb} , minimization of the gearbox volume V_{gb} , and maximization of the gearbox efficiency η_{gb} . For implementation in the optimization algorithm, the efficiency objective is transformed into a minimization form by considering its negative value. The objective vector is therefore defined as:

$$\text{Minimize } F(x) = [m_{gb}(x), V_{gb}(x), -\eta_{gb}(x)] \quad (29)$$

Accordingly, the design problem becomes a simultaneous three-objective minimization problem. The optimization procedure considers three continuous decision variables, namely the transmission ratio of the first stage u_1 , the face-width coefficient of the first stage X_{ba1} , and the face-width coefficient of the second stage X_{ba2} . These design parameters directly affect the geometric dimensions, structural compactness, mass characteristics, and power transmission efficiency of the gearbox system. To guarantee feasible and manufacturable gearbox configurations, the following design constraints are imposed according to [25]:

$$1 \leq u_i \leq 9 \quad (30)$$

$$0.25 \leq X_{bai} \leq 0.4 \quad (31)$$

3. METHODOLOGY

The proposed optimization framework was implemented in MATLAB using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). As formulated in Section 2.4, the optimization problem simultaneously considers three objectives: minimizing gearbox mass, minimizing gearbox volume, and maximizing gearbox efficiency. For implementation in NSGA-II, the maximization of efficiency was transformed into a minimization objective by using its negative value, as defined in Eq. (29). The design vector consists of three continuous variables: u_1 , X_{ba1} , and X_{ba2} . The lower and upper bounds of these variables follow the constraints given in Eqs. (30) and (31). In addition, the second-stage transmission ratio is calculated from the total transmission ratio as ($u_2 = u_h/u_1$). The u_h was varied from 5 to 35 with an interval of 5. For each value of u_h , an independent NSGA-II run was performed. The population size was set to 100 and the maximum number of generations was 300. The genetic operators included tournament selection, simulated binary crossover, and polynomial mutation. The crossover probability was 0.9, while the mutation probability was (1/3). The distribution indices for crossover and mutation were both set to 20. During each generation, candidate solutions were evaluated using the gearbox calculation model described in Section 2. The objective values include gearbox mass, gearbox volume, and gearbox efficiency. Infeasible solutions were penalized using infinite objective values. Additional technical limits were imposed on efficiency, gearbox mass, gearbox volume, gear tooth number, and positive geometric dimensions to ensure physically meaningful designs. The convergence and diversity of the Pareto fronts were evaluated using hypervolume, spacing, and spread metrics. For hypervolume calculation, fixed normalization was applied using predefined ideal and nadir points, and a Monte Carlo sampling approach with 50,000 sample points was used. The number of Pareto solutions was also recorded during the optimization process. After completing all NSGA-II runs, the final non-dominated solutions were collected to construct global Pareto maps. Representative solutions were then selected for each value of u_h , including the maximum-efficiency solution, minimum-mass solution, minimum-volume solution, and a compromise solution. The compromise solution was determined using a combined performance score defined as the ratio of efficiency to the product of gearbox mass and gearbox volume.

4. RESULTS AND DISCUSSION

The proposed NSGA-II framework was applied to optimize the gearbox under different u_h (from 5 to 35). Three conflicting objectives were simultaneously considered, including minimization of gearbox mass, minimization of gearbox volume, and maximization of

gearbox efficiency. The obtained Pareto-optimal solutions provide important insights into the influence of transmission ratio on gearbox compactness and transmission performance.

Figure 2 illustrates the global Pareto distribution between gearbox mass and transmission efficiency. It can be observed that the transmission ratio has a strong influence on the Pareto front characteristics. Lower u_h (5 to 10) tend to provide higher efficiencies, typically above 97%, but these solutions are associated with larger gearbox masses. In contrast, higher transmission ratios produce more compact gearbox structures with reduced mass, although the corresponding efficiency gradually decreases. This behaviour indicates a clear trade-off between lightweight design and transmission efficiency.

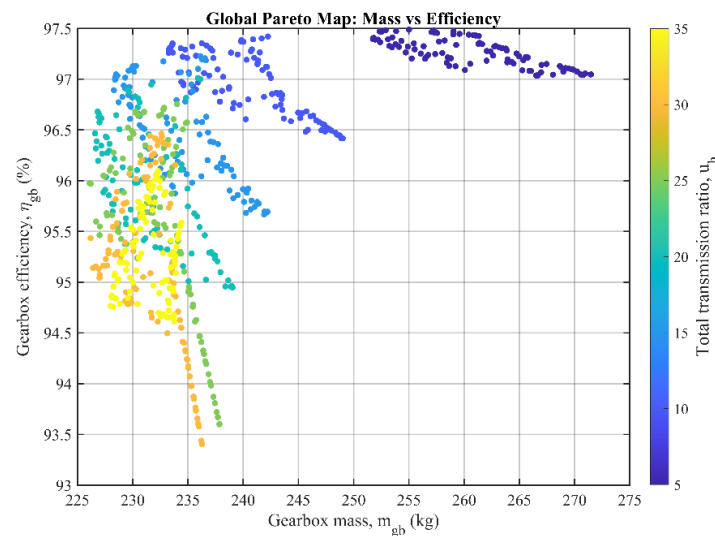


Fig. 2. Global Pareto map showing the relationship between gearbox mass and gearbox efficiency for different total transmission ratios

A similar tendency can be observed in Fig. 3, which presents the relationship between gearbox volume and efficiency. As the total transmission ratio increases, the gearbox volume generally increases while the efficiency decreases. The Pareto fronts become more dispersed for larger values of u_h , indicating stronger conflicts among the three objectives. In particular, the lowest gearbox volumes are obtained for $u_h=5$, whereas the largest volumes appear at $u_h=35$. The influence of the transmission ratio on the best objective values is summarized in Fig. 4. The maximum gearbox efficiency continuously decreases from approximately 97.5% at $u_h=5$ to about 96.1% at $u_h=35$. Meanwhile, the minimum gearbox mass rapidly decreases from approximately 271 kg to around 234 kg as u_h increases from 5 to 25, after which the reduction becomes insignificant. On the other hand, the minimum gearbox volume increases almost monotonically with increasing transmission ratio. These results demonstrate that lower transmission ratios are favourable for high-efficiency operation, whereas moderate transmission ratios provide more compact gearbox designs. Figure 5 compares the efficiencies of several representative solutions, including the maximum-efficiency solution, minimum-mass solution, minimum-volume solution, and compromise solution. As expected, the maximum-efficiency solutions always exhibit the highest efficiencies over the entire transmission-ratio range. However, the compromise solutions provide more balanced

performance by simultaneously maintaining relatively low mass and moderate volume while preserving acceptable efficiency levels. The efficiency of the compromise solutions varies between approximately 94.6% and 97.0%.

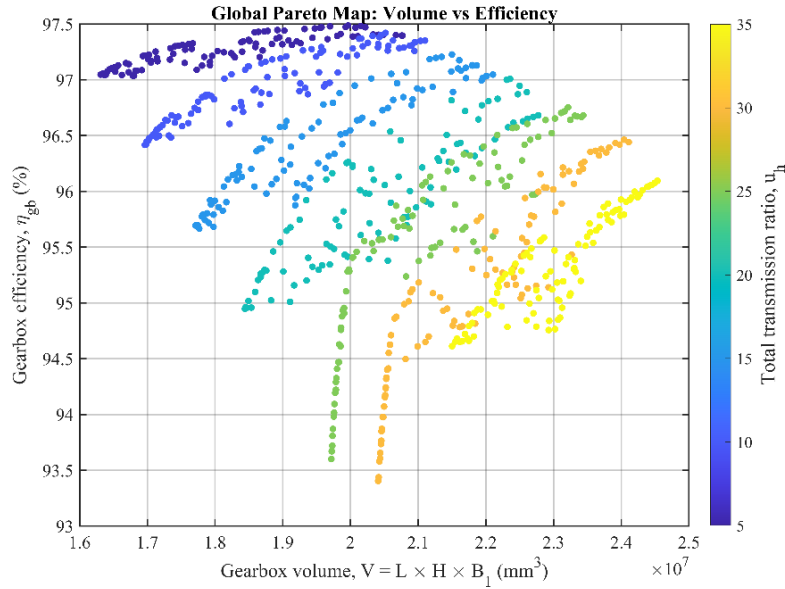


Fig. 3. Global Pareto map showing the relationship between gearbox volume and gearbox efficiency under different transmission ratios

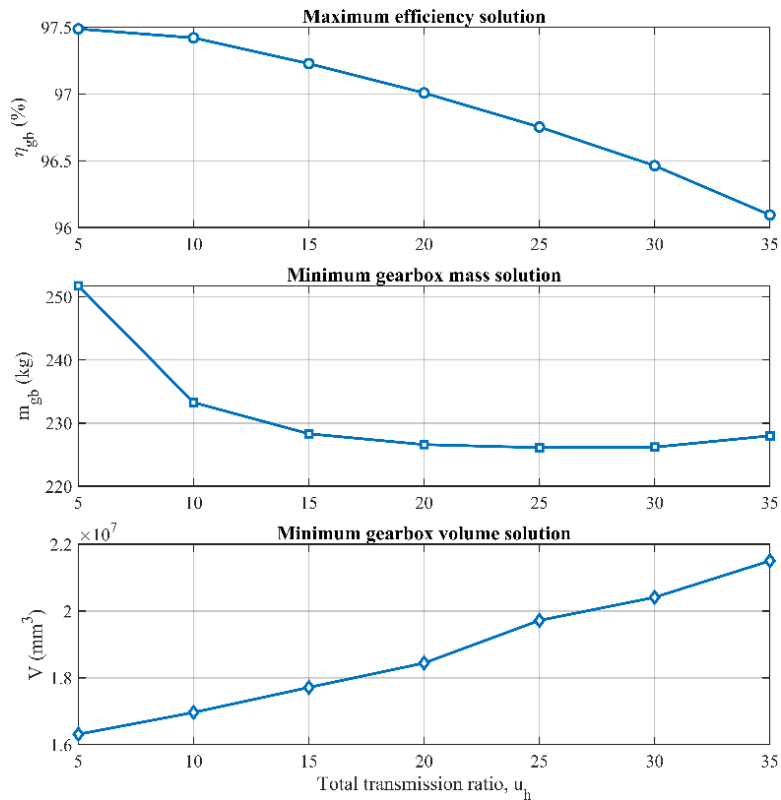


Fig. 4. Effect of the total transmission ratio on the maximum-efficiency solution, minimum-mass solution, and minimum-volume solution

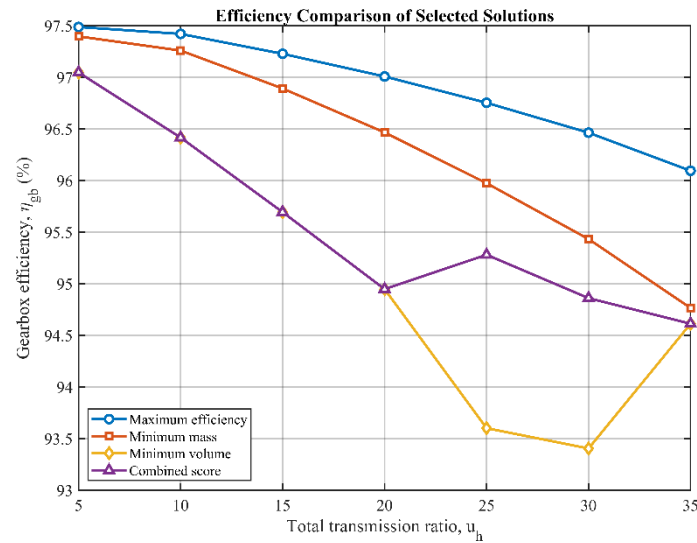


Fig. 5. Efficiency comparison among representative Pareto-optimal solutions

The corresponding transmission-ratio distributions of the selected compromise solutions are shown in Fig. 6. The u_1 generally increases with increasing of u_h , while u_2 also increases but at a lower rate. This indicates that the optimization algorithm tends to allocate a larger portion of the transmission ratio to the first stage when the total transmission ratio becomes large. Such a distribution helps improve geometric compactness and maintain stable efficiency characteristics.

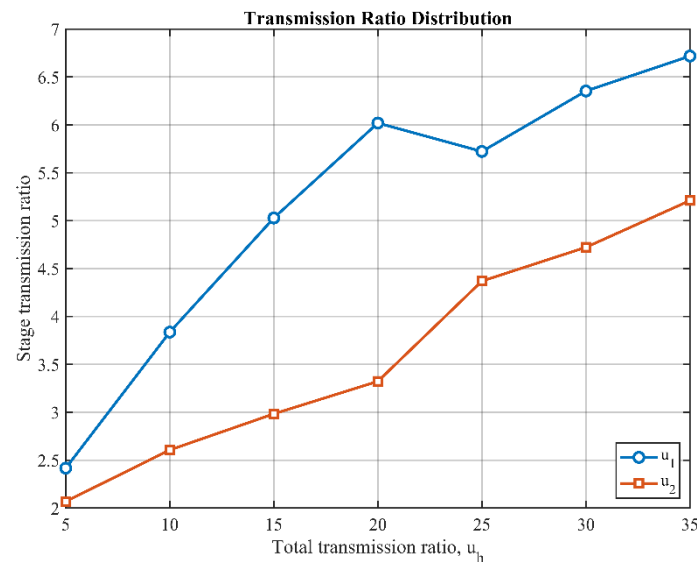


Fig. 6. Distribution of first-stage and second-stage transmission ratios for the selected compromise solutions

Figure 7 presents the global three-dimensional Pareto front considering gearbox mass, gearbox volume, and gearbox efficiency simultaneously. The obtained Pareto surface clearly demonstrates the conflicting relationships among the three objectives. Solutions with high efficiency are mainly concentrated in the region of large gearbox mass and relatively small gearbox volume, whereas lightweight solutions are generally associated with lower

efficiencies. The selected compromise solutions projected onto the mass-efficiency plane are shown in Fig. 8. These solutions are well distributed along the global Pareto front, confirming the capability of the proposed framework to identify representative trade-off designs for different transmission ratios. In practical engineering applications, such solutions can provide designers with flexible alternatives depending on design priorities.

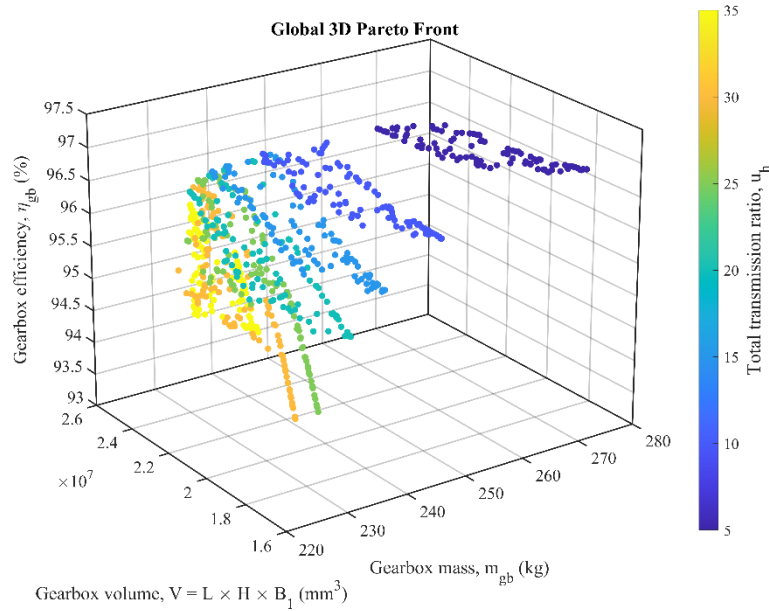


Fig. 7. Global three-dimensional Pareto front considering gearbox mass, gearbox volume, and gearbox efficiency

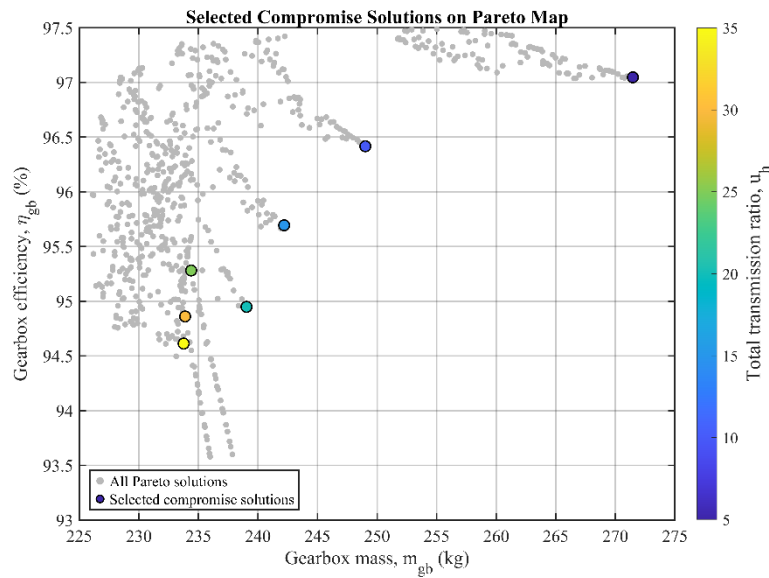


Fig. 8. Selected compromise solutions projected onto the global Pareto map

Figure 9 shows the compactness trend of the selected compromise solutions. As the total transmission ratio increases, the gearbox mass decreases significantly, while the gearbox volume increases continuously. This trend further confirms the existence of a design conflict between lightweight construction and volumetric compactness. Finally, the normalized

performances of the selected compromise solutions are presented in Fig. 10. The normalized comparison clearly reveals that no single solution simultaneously achieves the best values for all objectives. Therefore, multi-objective optimization is essential for balancing gearbox efficiency, structural compactness, and lightweight design requirements.

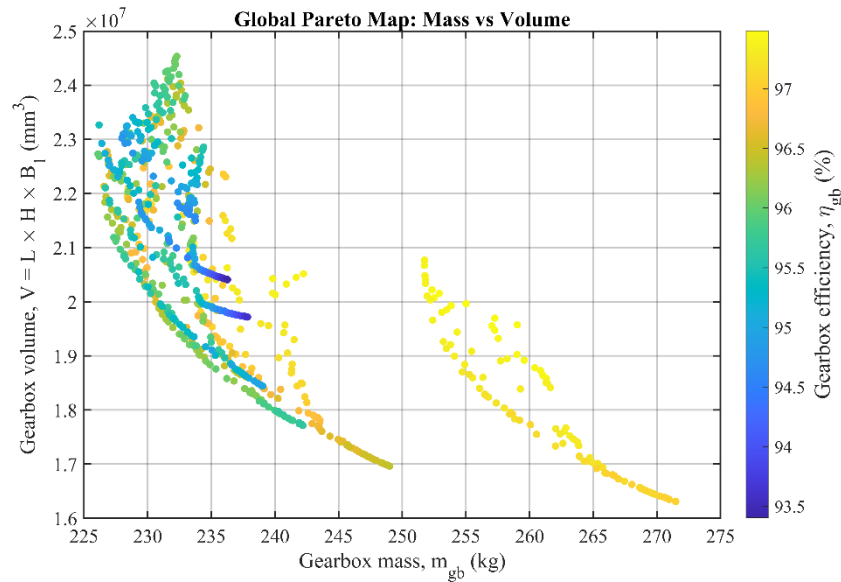


Fig. 9. Compactness trend of the selected compromise solutions with increasing total transmission ratio

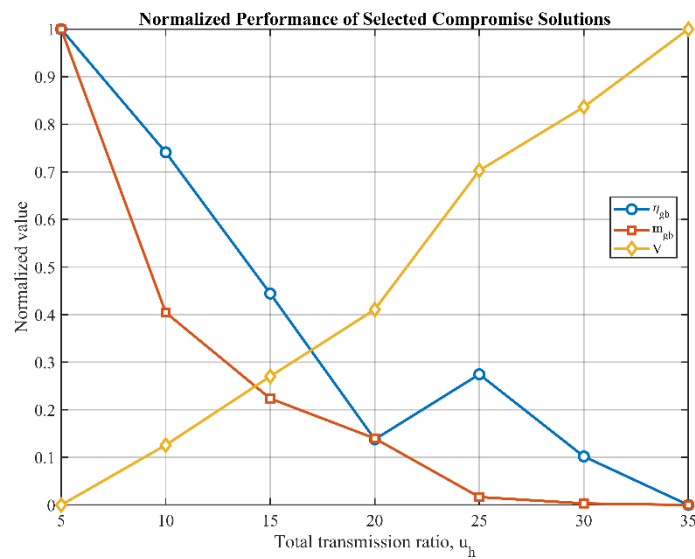


Fig. 10. Normalized performance comparison of the selected compromise solutions

5. CONCLUSION

Overall, the obtained results confirm that the proposed NSGA-II-based framework is effective for solving the multi-objective optimization problem of two-stage helical gearboxes namely in simultaneous optimization in case of three conflicting objectives: minimizing

gearbox mass, minimizing gearbox volume, and maximizing transmission efficiency. The optimization results also demonstrate that the total transmission ratio significantly affects the Pareto-optimal design characteristics and should therefore be carefully considered during gearbox design. The quantitative analysis and findings specified a significant novelty of the study.

This study proposed a multi-objective optimization framework for two-stage helical gearboxes using the NSGA-II algorithm. Three conflicting objectives, including minimization of gearbox mass, minimization of gearbox volume, and maximization of gearbox efficiency, were simultaneously optimized. The influence of u_h was systematically investigated within the range of $u_h = 5\text{--}35$. The results showed that the transmission ratio strongly affects gearbox compactness and efficiency. Lower transmission ratios provided higher efficiencies, reaching approximately 97.5%, but resulted in larger gearbox masses. In contrast, higher transmission ratios reduced gearbox mass while increasing gearbox volume and decreasing efficiency. The minimum gearbox mass decreased from about 271 kg to approximately 234 kg as u_h increased from 5 to 35, whereas the minimum gearbox volume increased from approximately $1.63 \times 10^7 \text{ mm}^3$ to $2.16 \times 10^7 \text{ mm}^3$. The obtained Pareto fronts demonstrated clear trade-offs among the three objectives, indicating that no single solution can simultaneously achieve the best values for all performance criteria. In addition, the optimal transmission-ratio distribution changed significantly with increasing total transmission ratio, highlighting its important role in gearbox design. It would be better to role the lubrication volume and the impact of the design in terms of the production costs or the operating conditions.

REFERENCES

- [1] WANG H., WANG H.-P., 1994, *Optimal Engineering Design of Spur Gear Sets*, Mech. Mach. Theory, 29/7, 1071–1080.
- [2] LEI Y., HOU L., FU Y., HU J., CHEN W., 2020, *Research on Vibration and Noise Reduction of Electric Bus Gearbox Based on Multi-Objective Optimization*, Applied Acoustics, 158, 107037.
- [3] PATIL M., RAMKUMAR P., SHANKAR K., 2017, *Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects*, J. Frict. Wear, 38/5, 430–436.
- [4] YOUNES E.B., CHANGENET C., BRUYERE J., RIGAUD E., PERRET-LIAUDET J., 2022, *Multi-Objective Optimization of Gear Unit Design to Improve Efficiency and Transmission Error*, Mechanism and Machine Theory, 167, 104499.
- [5] CHONG T.-H., 2020, *A Design Method of Gear Trains Using a Genetic Algorithm*, International Journal of Precision Engineering and Manufacturing, 1/1, 62–70.
- [6] COELLO C.A., VAN VELDHUIZEN D.A., LAMONT G.B., 2007, *Evolutionary Algorithms for Solving Multi-Objective Problems*, 2nd ed. New York, Springer.
- [7] KONAK A., COIT D.W., SMITH A.E., 2006, *Multi-Objective Optimization Using Genetic Algorithms: a Tutorial*, Reliab. Eng. Syst. Saf., 91/9, 992–1007.
- [8] DEB K., PRATAP A., AGARWAL S., MEYARIVAN T., 2002, *A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II*, IEEE Trans. Evol. Comput., 6/2, 182–197.
- [9] MENDEZ M., ROSSIT D.A., GONZALEZ B., FRUTOS M., 2020, *Proposal and Comparative Study of Evolutionary Algorithms for Optimum Design of a Gear System*, IEEE Access, 8, 3482–3497.
- [10] PATIL M., RAMKUMAR P., KRISHNAPILAI S., 2017, *Multi-Objective Optimization of Two Stage Spur Gearbox Using NSGA-II*, SAE Technical Paper, 0148–7191.
- [11] XIN W., ZHANG Y., FU Y., YANG W., ZHENG H., 2023, *A Multi-Objective Optimization Design Approach of Large Mining Planetary Gear Reducer*, Scientific Reports, 13/1, 18640.

- [12] KUMAR A., RAMKUMAR P., SHANKAR K., 2024, *Multi-Objective 3-Stage Wind Turbine Gearbox (WTG) with Tribological Constraint*, Mechanics Based Design of Structures and Machines, 52/8, 5263–5289.
- [13] HU M., ZHU J., GONG L., LU Z., LIU H., 2025, *Multi-Objective Optimization of an Aeroengine Accessory Gearbox Transmission Based on a Heuristic Algorithm*, Journal of Aerospace Engineering, 38/2 ,04024126.
- [14] QI L., ZHOU J., XU H., 2022, *Multi-Objective Optimization of Gearbox Based on Panel Acoustic Participation and Response Surface Methodology*, Journal of Low Frequency Noise, Vibration and Active Control, 41/3, 1108–1130.
- [15] SANGHVI R., VASHI A., PATOLIA H., JIVANI R., 2014, *Multi Objective Optimization of Two Stage Helical Gear Train Using NSGA II*, Journal of optimization, 2014/1, 670297.
- [16] LE X.-H., VU N.-P., 2023, *Multi-Objective Optimization of a Two-Stage Helical Gearbox Using Taguchi Method and Grey Relational Analysis*, Appl. Sci., 13/13, 7601.
- [17] DINH V.-T., et al., 2024, *Multi-Objective Optimization of a Two-Stage Helical Gearbox with Second Stage Double Gear-Sets Using TOPSIS Method*, Processes, 12/6, 1160.
- [18] DINH V.-T., et al., 2024, *Multi-Objective Optimization of a Two-Stage Helical Gearbox Using MARCOS Method*, Designs, 8/3, 53.
- [19] VU D.-B., TRAN H.-D., DINH V.-T., VU D., VU N.-P., NGUYEN V.-T., 2024, *Solving a Multi-Objective Optimization Problem of a Two-Stage Helical Gearbox with Second-Stage Double Gear Sets Using the MAIRCA Method*, Applied Sciences, 14/12, 5274.
- [20] DINH V.-T., TRAN H.-D., VU D.-B., VU D., VU N.-P., LUU T.A., 2024, *Using TOPSIS Method for Solving Multi-Objective Optimization of a Two-Stage Helical Gearbox with First Stage Double Gear-Sets*, Journal of Machine Engineering, 24/4, 41–56.
- [21] HUNG T.Q., HUONG T.T.T., 2023, *Multi-Objective Optimization of a Two-Stage Helical Gearbox to Improve Efficiency and Reduce Height*, Journal of Environmental Science and Engineering, B12, 236–247.
- [22] VAN THANH D., DANH B.T., VAN CHIEN THANG N., DANH T.H., VAN TRANG N., TU H.X., 2023, *Multi-Objective Optimization of a Two-Stage Helical Gearbox with Second Stage Double Gear Sets to Reduce Gearbox Length and Increase Gearbox Efficiency*, in International Conference on Engineering Research and Applications, Springer, 229–244.
- [23] HUNG T.Q., BINH V.D., VAN THANH D., TUNG L.A., TUAN N.K., 2025, *Multi-Objective Optimization of a Two-Stage Helical Gearbox with Two Gear Sets in First Stage to Reduce Volume and Enhance Efficiency Using the EAMR Technique*, Engineering, Technology & Applied Science Research, 15/1, 9288–19294.
- [24] BINH V.D., VAN THANH D., NGUYEN K.M., HUNG L.X., 2024, *Multi-Objective Optimization of a Two-Stage Helical Gearbox with Double Gears in the First Stage Using MARCOS*, Engineering, Technology & Applied Science Research, 14/6, 18245–18251.
- [25] CHAT T., VAN UYEN L., 2007, *Design and Calculation of Mechanical Transmissions Systems*, Educational Republishing House: Hanoi, Vietnam 1.
- [26] JELASKA D.T., 2012, *Gears and Gear Drives*, John Wiley & Sons: Hoboken, NJ, USA.